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SYNCHRONIZATION OF A CLASS OF DELAYED CHAOTIC NEURAL NETWORKS WITH FULLY UNKNOWN PARAMETERS[1]

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Abstract. This paper presents a global asymptotic synchronization scheme for a class of delayed chaotic neural networks when the parameters of the drive system are fully unknown and different from those of the response system. Using the Lyapunov stability theory and the inverse optimal control approach, an adaptive synchronization controller is proposed to guarantee the global asymptotic synchronization of state trajectories for two delayed chaotic neural networks with fully unknown parameters. The present controller can easily be implemented in practice. An illustrative example is used to demonstrate the effectiveness of the present method.
Keywords. Delayed chaotic neural networks; unknown parameters; synchronization; inverse optimal control; Lyapunov theory
AMS (MOS) subject classification: 0545 TP273

1 Introduction

Over the last decades, synchronization of chaotic systems has been intensively investigated by many researchers. Since chaos synchronization has potential applications in several areas such as secure communication [1–3], chemical reactions, biological systems, information science, etc., many different chaos synchronization strategies have been developed, including drive-response control [4], coupling control [5], variable structure control [6], adaptive control [7], impulsive control [8, 9], active control [10–12]. Nevertheless, in the aforementioned methods and many other existing synchronization methods, one major difficulty seems to be caused by the requirement

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of solving some associated partial differential equations. In order to alleviate this computational problem, the inverse optimal control theory has been developed recently in nonlinear systems [13].

Recently, there is increasing interest in the study of dynamical properties of delayed neural networks (DNNs). Most previous studies have predominantly concentrated on the stability analysis and periodic oscillations of this kind of networks [14–16]. It has been shown that such networks can exhibit some complicated dynamics and even chaotic behaviors. In particular, by appropriately choosing the network parameters and time delays, the dynamical behavior of these networks can be made much complicated [17–19]. However, there are few studies in the synchronization of this class of chaotic neural networks with or without delays [20]. In [20], the parameters of the two neural networks to be synchronized are known and are identical. Because system parameters are inevitably perturbed by external factors and cannot be exactly known a priori, synchronization of two delayed chaotic neural networks with fully unknown parameters is more essential and useful in real world applications.

In this paper, we study the global asymptotic synchronization of a class of delayed chaotic neural networks with fully unknown parameters. The method for the controller and the parameter adaptation is designed by the inverse optimal control approach and the Lyapunov stability theory.

The paper is organized as follows. In Section 2, the problem considered in this paper is described. In Section 3, an adaptive synchronization controller that can globally asymptotically synchronize two delayed chaotic neural networks are designed. In Section 4, a simulation result is shown. In Section 5, the paper is concluded with a few remarks.

2 Problem Description

A class of delayed chaotic neural networks in this paper is described by the following differential equations:

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} g_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(x_j(t-\tau_j)) + U_i \tag{1}$$

where n denotes the number of neurons in the network, x_i denotes the state variable associated with the ith neuron, $\tau_j \geq 0$ denotes bounded delay and $\rho = \max(\tau_j)$, $c_i > 0$, a_{ij} indicates the strength of the neuron interconnections within the network, b_{ij} indicates the strength of the neuron interconnections within the network with constant delay parameter τ_j, $j = 1 \cdots n$, and U_i is a constant input vector function. The activation function $g_j(x_j)$ satisfies:

$$0 \leq \frac{g_j(\xi) - g_j(\zeta)}{\xi - \zeta} \leq \delta_j, \forall \xi, \zeta \in \Re \text{ and } \xi \neq \zeta.$$

The initial conditions of system (1) are given by $x_i(t) = \phi_i(t) \in C([-\rho, 0], \Re)$, where $C([-\rho, 0], \Re)$ denotes the set of all continuous functions from $[-\rho, 0]$ to $\Re$.

Chaotic dynamics is extremely sensitive to initial conditions. Even infinitesimal changes in the initial condition will lead to an exponential divergence of orbits. In order to observe the synchronization behavior in this class of delayed chaotic neural networks, we study two chaotic neural networks where the drive system and the response system have identical dynamic structure but with different parameters. The drive system's state variables are denoted by x_i and the response system's state variable are denoted by z_i. Suppose that the parameters of the drive system are unknown and uncertain, and the response system is described by the following equations:

$$\dot{z}_i(t) = -\tilde{c}_i z_i(t) + \sum_{j=1}^{n} \tilde{a}_{ij} g_j(z_j(t)) + \sum_{j=1}^{n} \tilde{b}_{ij} g_j(z_j(t-\tau_j)) + U_i + u_i(t) \tag{2}$$

where the initial conditions of system (2) are given by $z_i(t) = \varphi_i(t) \in C([-\rho, 0], \Re)$, $\tilde{c}_i$, $\tilde{a}_{ij}$ and $\tilde{b}_{ij}$ are parameters of the response system which need to be estimated, and $u_i(t)$ denotes the designed controller that will realize the synchronization of system (1) and system (2). The goal of the controller design is to obtain $u_i(t)$ so that $\lim_{t\to\infty} (z_i(t) - x_i(t)) = 0$, $i = 1, \cdots n$.

3 Controller Design

Let us define the synchronization error signal as $e_i(t) = z_i(t) - x_i(t)$, where $x_i(t)$ and $z_i(t)$ are the ith state variable of the drive and response neural networks, respectively. $e_i(t) \to 0$ as $t \to \infty$ means that the drive neural network and the response neural network are synchronized. Therefore, the error dynamics between (1) and (2) can be expressed by

$$\begin{aligned} \dot{e}_i(t) = &- c_i e_i(t) + \sum_{j=1}^{n} a_{ij} f_j(e_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(e_j(t-\tau_j)) \\ &+ (c_i - \tilde{c}_i) z_i(t) + \sum_{j=1}^{n} (\tilde{a}_{ij} - a_{ij}) g_j(z_j(t)) \\ &+ \sum_{j=1}^{n} (\tilde{b}_{ij} - b_{ij}) g_j(z_j(t-\tau_j)) + u_i(t) \end{aligned} \tag{3}$$

where $f_j(e_j(t)) = g_j(e_j(t) + x_j(t)) - g_j(x_j(t))$ and $f_j(0) = 0$, or by the following compact form

$$\begin{aligned} \dot{e}(t) = &- Ce(t) + Af(e(t)) + Bf(e(t-\tau)) + \hat{C}z(t) + \hat{A}g(z(t)) \\ &+ \hat{B}g(z(t-\tau)) + u(t) \end{aligned} \tag{4}$$

where $C = \text{diag}(c_1 \ \cdots c_n)$, $A = [a_{ij}]_{n\times n}$, $B = [b_{ij}]_{n\times n}$, $\tau = (\tau_1 \ \cdots, \ \tau_n)^T$, $\hat{C} = \text{diag}\,[c_i - \tilde{c}_i]_{n\times n} = \text{diag}\,[\hat{c}_i]_{n\times n}$, $\hat{A} = [\tilde{a}_{ij} - a_{ij}]_{n\times n} = [\hat{a}_{ij}]_{n\times n}$, $\hat{B} = [\tilde{b}_{ij} - b_{ij}]_{n\times n} = [\hat{b}_{ij}]_{n\times n}$, $f(e(t)) = (f_1(e_1(t)), \ldots, f_n(e_n(t)))^T$, $f(e(t-\tau)) = (f_1(e_1(t-\tau_1)), \ldots, f_n(e_n(t-\tau_n)))^T$, and $u(t) = (u_1(t), \ldots, u_n(t))^T$.

Lemma1 [21]. For all matrices X, $Y \in R^{n\times k}$ and $Q \in R^{n\times n}$ with $Q = Q^T > 0$, the following inequality holds:

$$X^T Y + Y^T X \leq X^T Q X + Y^T Q^{-1} Y. \tag{5}$$

Theorem 1. For system (4), if the controller is chosen as

$$u(t) = -\left(AA^T + BB^T + 2\Sigma^2\right) e\,(t) \tag{6}$$

and the parameter adaptation update law is chosen as

$$\begin{array}{l} \dot{\hat{c}}_i = e_i\,(t)\,z_i\,(t)\,, \\ \dot{\hat{a}}_{ij} = -e_i\,(t)\,g_j\,(z_j\,(t))\,, \\ \dot{\hat{b}}_{ij} = -e_i\,(t)\,g_j\,(z_j\,(t-\tau_j))\,, \end{array} \tag{7}$$

$i, j = 1, \cdots, n$, then systems (1) and (2) will be globally asymptotically synchronized, where $\Sigma = \text{diag}(\delta_1, \ldots, \delta_n)$.

Proof:

Define

$$\varepsilon \triangleq \left[e^T(t), \hat{c}_1, \cdots, \hat{c}_n, \hat{a}_{11}, \cdots, \hat{a}_{1n}, \cdots, \hat{a}_{n1}, \cdots, \hat{a}_{nn}, \hat{b}_{11}, \cdots, \hat{b}_{1n}, \cdots, \hat{b}_{n1}, \cdots, \hat{b}_{nn}\right]^T.$$

We choose

$$\begin{aligned} V\,(\varepsilon) = {} & \frac{1}{2} e^T\,(t)\,e\,(t) + \frac{1}{2}\sum_{j=1}^{n} \int_{t-\tau_j}^{t} \delta_j^2 e_j^2\,(s)\,ds \\ & + \frac{1}{2}\sum_{i=1}^{n} \hat{c}_i^2 + \frac{1}{2}\sum_{i=1,j=1}^{n} \hat{a}_{ij}^2 + \frac{1}{2}\sum_{i=1,j=1}^{n} \hat{b}_{ij}^2. \end{aligned} \tag{8}$$

Its time-derivative can be derived as follows:

$$\begin{aligned} \dot{V}\,(\varepsilon) = {} & e^T\,(t)\,\dot{e}\,(t) + \frac{1}{2} e^T\,(t)\,\Sigma^2 e\,(t) - \frac{1}{2} e^T\,(t-\tau)\,\Sigma^2 e\,(t-\tau) \\ & + \sum_{i=1}^{n} \hat{c}_i \dot{\hat{c}}_i + \sum_{i=1,j=1}^{n} \hat{a}_{ij}\dot{\hat{a}}_{ij} + \sum_{i=1,j=1}^{n} \hat{b}_{ij}\dot{\hat{b}}_{ij} \\ = {} & - e^T\,(t)\,Ce(t) + e^T(t)\big(Af(e(t)) + Bf(e(t-\tau))\big) + e^T\,(t)\,u(t) \\ & + e^T\,(t)\left(\hat{c}z\,(t) + \hat{A}g\,(z\,(t)) + \hat{B}g\,(z\,(t-\tau))\right) + \frac{1}{2} e^T\,(t)\,\Sigma^2 e\,(t) \\ & - \frac{1}{2} e^T\,(t-\tau)\,\Sigma^2 e\,(t-\tau) - \sum_{i=1}^{n} \hat{c}_i \dot{\hat{c}}_i + \sum_{i=1,j=1}^{n} \hat{a}_{ij}\dot{\hat{a}}_{ij} + \sum_{i=1,j=1}^{n} \hat{b}_{ij}\dot{\hat{b}}_{ij} \\ \triangleq {} & L_{\bar{f}} V + (L_g V)\,u\,(t) \end{aligned} \tag{9}$$

where

$$\begin{aligned} L_{\bar{f}} V \triangleq & - e^T(t) Ce(t) + e^T(t)\left(Af(e(t)) + Bf(e(t-\tau))\right) \\ & + e^T(t)\Big(\hat{c}z(t) + \hat{A}g(z(t)) + \hat{B}g(z(t-\tau))\Big) + \frac{1}{2}e^T(t)\Sigma^2 e(t) \\ & - \frac{1}{2}e^T(t-\tau)\Sigma^2 e(t-\tau) - \sum_{i=1}^{n}\hat{c}_i\dot{\tilde{c}}_i + \sum_{i=1,j=1}^{n}\hat{a}_{ij}\dot{\tilde{a}}_{ij} + \sum_{i=1,j=1}^{n}\hat{b}_{ij}\dot{\tilde{b}}_{ij} \end{aligned} \tag{10}$$

and

$$L_g V \triangleq e^T(t). \tag{11}$$

Applying Lemma 1 with $Q = I$, we get

$$\begin{aligned} \dot{V}(\varepsilon) \leq & - e^T(t) Ce(t) + \frac{1}{2}e^T(t) AA^T e(t) + \frac{1}{2}f^T(e(t))f(e(t)) \\ & + \frac{1}{2}e^T(t) BB^T e(t) + \frac{1}{2}f^T(e(t-\tau))f(e(t-\tau)) \\ & + \frac{1}{2}e^T(t)\Sigma^2 e(t) - \frac{1}{2}e^T(t-\tau)\Sigma^2 e(t-\tau) + e^T(t)u(t) \\ & + e^T(t)\Big(\hat{c}z(t) + \hat{A}g(z(t)) + \hat{B}g(z(t-\tau))\Big) \\ & - \sum_{i=1}^{n}\hat{c}_i\dot{\tilde{c}}_i + \sum_{i=1,j=1}^{n}\hat{a}_{ij}\dot{\tilde{a}}_{ij} + \sum_{i=1,j=1}^{n}\hat{b}_{ij}\dot{\tilde{b}}_{ij} \\ \leq & - e^T(t) Ce(t) + \frac{1}{2}e^T(t) AA^T e(t) + \frac{1}{2}e^T(t)\Sigma^2 e(t) \\ & + \frac{1}{2}e^T(t) BB^T e(t) + \frac{1}{2}e^T(t-\tau)\Sigma^2 e(t-\tau) \\ & + \frac{1}{2}e^T(t)\Sigma^2 e(t) - \frac{1}{2}e^T(t-\tau)\Sigma^2 e(t-\tau) + e^T(t)u(t) \\ & + \sum_{i=1}^{n}\hat{c}_i\left(e_i(t)z_i(t) - \dot{\tilde{c}}_i\right) + \sum_{i=1,j=1}^{n}\hat{a}_{ij}\left(\dot{\tilde{a}}_{ij} + e_i(t)g_j(z_j(t))\right) \\ & + \sum_{i=1,j=1}^{n}\hat{b}_{ij}\left(\dot{\tilde{b}}_{ij} + e_i(t)g_j(z_j(t-\tau_j))\right). \end{aligned}$$

Substituting (7) into the above equality, we obtain

$$\dot{V}(\varepsilon) \leq e^T(t)\left(-C + \frac{1}{2}AA^T + \frac{1}{2}BB^T + \Sigma^2\right)e(t) + e^T(t)u(t). \tag{12}$$

If we denote

$$R^{-1}(\varepsilon) = \frac{1}{\beta}\left(AA^T + BB^T + 2\Sigma^2\right) \tag{13}$$

where $\beta \geq 2$ is a constant, we have

$$u = -\beta R(\varepsilon)^{-1}(L_g V)^T = -\left(AA^T + BB^T + 2\Sigma^2\right)e(t). \tag{14}$$

Substituting (14) in (12), we obtain

$$\begin{aligned}\dot{V}(\varepsilon) &\le -e^T(t)\left(C+\frac{1}{2}AA^T+\frac{1}{2}BB^T+\Sigma^2\right)e(t)\\ &\le -\lambda_{\min}\left(C+\frac{1}{2}AA^T+\frac{1}{2}BB^T+\Sigma^2\right)\|e\|^2\\ &\le 0\end{aligned} \tag{15}$$

which implies $\dot{V} < 0$ for all $e \neq 0$.This means that the proposed controller (14) can globally asymptotically synchronize the system (1) and the system (2). This completes the proof of the theorem.

The optimal stabilization guarantees several desirable properties for closed-loop systems, including stability margins. In a direct approach we would have to solve the Hamilton-Jacobi-Bellman (HJB) equation, which is not an easy task. Besides, the achieved robustness is largely independent of the particular choice of two functions, denoted $l(\varepsilon) > 0$ and $R(\varepsilon) > 0$. But in the inverse approach, a stabilizing feedback is designed first and then shown to optimize a cost functional of the following form [13]

$$J(u) = \lim_{t\to\infty}\left\{2\beta V(\varepsilon(t)) + \int_0^t \left(l(\varepsilon(\tau)) + u^T R(\varepsilon(\tau))u\right)d\tau\right\}, \quad \beta \ge 2. \tag{16}$$

The stabilizing feedback problem of system (4) has been solved under the control of (6) and (7). So we need to find a positive real-valued function $R(\varepsilon)$ and a positive definite function $l(\varepsilon)$, such that the cost functional $J(u)$ is minimized.

Theorem 2. If we choose

$$l(\varepsilon) = -2\beta L_{\bar{f}}V + \beta^2 (L_gV) R^{-1}(\varepsilon)(L_gV)^T \tag{17}$$

and

$$R(\varepsilon) = \beta\left(AA^T + BB^T + 2\Sigma^2\right)^{-1}, \beta \ge 2 \tag{18}$$

the cost functional (16) for system (4) under the parameter update laws (7) and the state feedback (6) will be minimized.

Proof: Based on the basic idea of the inverse optimal control theory, we will prove that $R(\varepsilon)$ is symmetric and positive definite, and $l(\varepsilon)$ is radically unbounded, i.e., $R(\varepsilon) = R^T(\varepsilon) > 0$ and $l(\varepsilon) > 0$ for all $\varepsilon \neq 0$ and $l(\varepsilon) \to +\infty$ as $\varepsilon \to \infty$. It is clear that $R(\varepsilon)$ chosen according to (18) satisfies the requirement. Using (6), (7), (10), (11) and (18), we obtain

$$\begin{aligned}l(\varepsilon) =& -2\beta e^T(t)\left(-C+\frac{1}{2}AA^T+\frac{1}{2}BB^T+\Sigma^2\right)e(t)\\ &+\beta e^T(t)\left(AA^T+BB^T+2\Sigma^2\right)e(t)\\ =&\ 2\beta e^T(t)Ce(t).\end{aligned} \tag{19}$$

This means that $l(\varepsilon)$ is radially unbounded. Substituting (14) into (9), we get

$$\dot{V}(\varepsilon) = L_{\bar{f}}V - \beta(L_gV)R^{-1}(\varepsilon)(L_gV)^T . \tag{20}$$

Multiplying it by -2β, we obtain

$$-2\beta\dot{V}(\varepsilon) = -2\beta L_{\bar{f}}V + 2\beta^2(L_gV)R^{-1}(\varepsilon)(L_gV)^T . \tag{21}$$

Considering (14) and (17), we get from (21)

$$l(\varepsilon) + u^T R(\varepsilon)u = -2\beta\dot{V}(\varepsilon) . \tag{22}$$

Substituting (22) into (16), we have

$$\begin{aligned} J(u) &= \lim_{t\to\infty}\left\{2\beta V(\varepsilon(t)) + \int_0^t \left(-2\beta\dot{V}(\varepsilon(\tau))\right)d\tau\right\} \\ &= \lim_{t\to\infty}\{2\beta V(\varepsilon(t)) + (-2\beta V(\varepsilon(t)) + 2\beta V(\varepsilon(0)))\} \\ &= 2\beta V(\varepsilon(0)) . \end{aligned} \tag{23}$$

Thus, the minimum of the cost function is $J_{\min}(u) = 2\beta V(\varepsilon(0))$ for the optimal control law (6) and (7). The theorem is proved.

4 Simulation

Consider a two-dimensional delayed cellular neural networks [18] model of the form (1), with:

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 1+\pi/4 & 20 \\ 0.1 & 1+\pi/4 \end{bmatrix},$$

$$B = \begin{bmatrix} -1.3\sqrt{2}\pi/4 & 0.1 \\ 0.1 & -1.3\sqrt{2}\pi/4 \end{bmatrix}, \quad \tau = 1,$$

and $g(x(t)) = 0.5(|x(t)+1| - |x(t)-1|)$. The chaotic behavior of the drive system is shown in Fig.1, which is plotted with the initial condition

$$[x_1(s),\ x_2(s)] = [0.01,\ \ 0.1]$$

for $-1 \le s \le 0$. The curves of the drive system's state x_1 and x_2 are shown in Fig.2.

In this simulation, the initial values of "unknown" parameter vectors of the response system are selected as:

$$\hat{C}(0) = \begin{bmatrix} 1.1 & 0 \\ 0 & 1.1 \end{bmatrix}, \quad \hat{A}(0) = \begin{bmatrix} 1.1+\pi/4 & 20.1 \\ 0.11 & 1.1+\pi/4 \end{bmatrix},$$

$$\hat{B}(0) = \begin{bmatrix} -1.29\sqrt{2}\pi/4 & 0.11 \\ 0.11 & -1.29\sqrt{2}\pi/4 \end{bmatrix}.$$

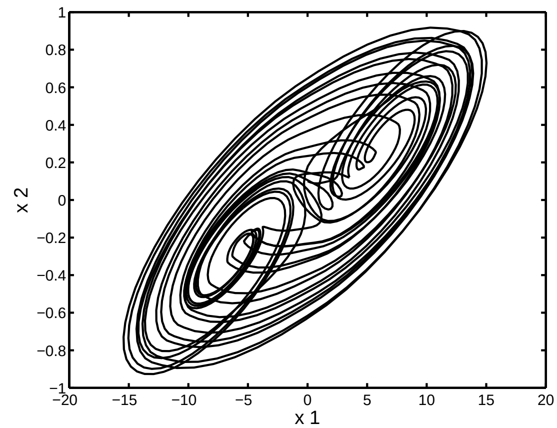

Figure 1: Chaotic behavior of the drive system

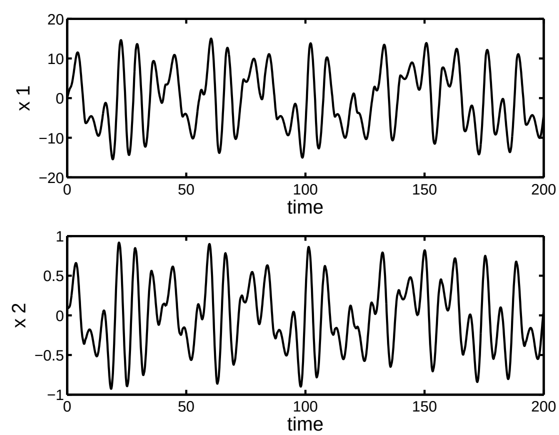

Figure 2: The curves of the drive system state x_1 and x_2

The chaotic behavior of the response system is shown in Fig.3, which is plotted with the initial condition

$$[z_1(s),\ z_2(s)] = [2,\ \ -0.2]$$

for $-1 \leq s \leq 0$. The synchronization error curves of the response system and the drive system are shown in Fig.4. The curves of $\tilde{C}$, $\tilde{A}$ and $\tilde{B}$ are shown, respectively, in Fig.5, Fig.6, and Fig.7. Obviously, the synchronization errors converge asymptotically to zero and the adaptive parameters converge asymptotically to some constants.

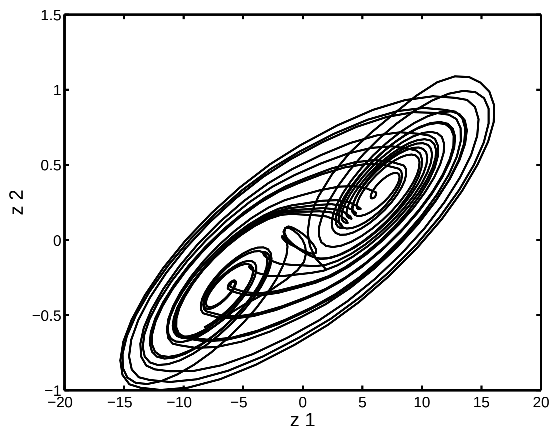

Figure 3: Chaotic behavior of the response system

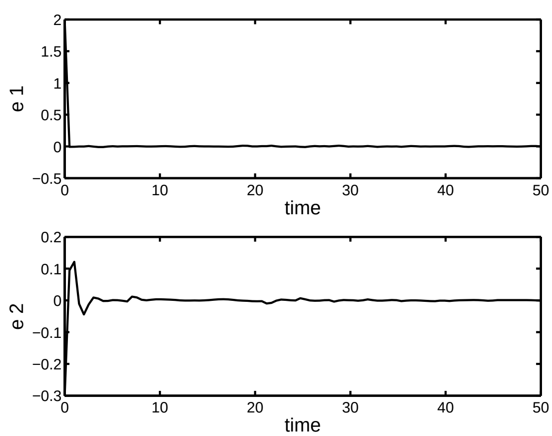

Figure 4: The error curves of the states z_i and x_i

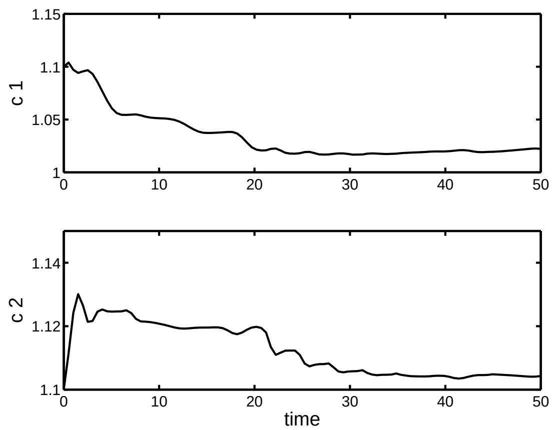

Figure 5: The curves of $\tilde{C}$

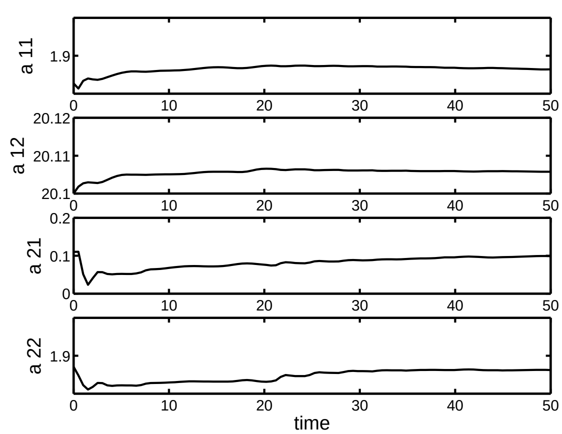

Figure 6: The curves of $\tilde{A}$

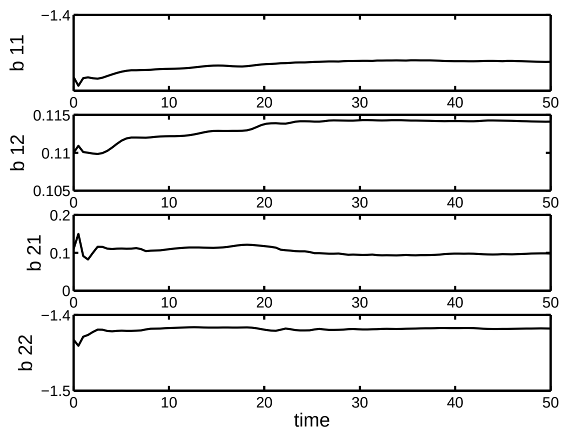

Figure 7: The curves of $\tilde{B}$

5 Conclusions

This paper deals with the global asymptotic synchronization problem for a class of delayed chaotic neural networks with fully unknown parameters. Using the Lyapunov stability theory and the inverse optimal control approach, an adaptive synchronization controller is proposed to guarantee the global asymptotic synchronization of state trajectories for two delayed chaotic neural networks with fully unknown parameters. The designed controller can easily be implemented in practice. An illustrative example is used to demonstrate the effectiveness of the presented method.

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