Delay-Dependent Guaranteed Cost Control for Uncertain Stochastic Fuzzy Systems With Multiple Time Delays

Huaguang Zhang, *Senior Member, IEEE*, Yingchun Wang, and Derong Liu, *Fellow, IEEE*

***Abstract*—This paper studies the guaranteed cost control problem for a class of uncertain stochastic nonlinear systems with multiple time delays represented by the Takagi–Sugeno fuzzy model with uncertain parameters. By constructing a new stochastic Lyapunov–Krasovskii functional, sufficient conditions for delay-dependent guaranteed cost control are obtained which do not require system transformation or relaxation matrices. Conditions for the existence of an optimal guaranteed cost controller are presented in the linear matrix inequality format. Simulation examples are provided to demonstrate the effectiveness of the proposed approach in this paper.**

***Index Terms*—Delay dependence, guaranteed cost control, linear matrix inequality (LMI), multiple time delays, stochastic fuzzy systems.**

I. Introduction

STABILITY analysis of stochastic systems has been well investigated in past years, since stochastic modeling has come to play an important role in many real systems, including nuclear processes, thermal processes, chemical processes, biology, socioeconomics, and immunology (see [16] and [25] for more details). Based on the Itô stochastic differential equation, many efforts have been devoted to extend the approaches from deterministic systems to stochastic systems (see, e.g., [8] and [13]). The Takagi–Sugeno (T-S) fuzzy modeling approach, which has been extensively studied for deterministic nonlinear systems (see [15], [18], [19], [22], and [30]), has also been applied to stochastic nonlinear systems (see, e.g., [5], [7], and [24]). On the other hand, time delays are often the source of instability and encountered in various engineering systems. Much attention has been devoted to the development of tools for stability analysis and controller design, and many results have been formulated [2], [4], [9], [14], [17], [21], [26], [29], [32]. These existing results for deterministic or stochastic systems can be divided into two categories: 1) delay-independent results [2], [21] and 2) delay-dependent results [4], [9], [14], [17], [26], [29], [32]. The former does not include any information on the sizes of delays, whereas the latter category employs such information and may be less conservative, particularly, when the sizes of delays are small. To obtain delay-dependent results, many approaches were developed for deterministic systems and stochastic ones. A descriptor system approach proposed in [9] was developed for stochastic systems [4], [32]. By transforming the original system into a descriptor system, the stability condition can be derived from analyzing the stability of such a descriptor system with a constrained Lyapunov matrix. The relaxation matrices were introduced for deterministic systems [14], [26] and stochastic ones [29] based on the Newton–Leibniz formula. This kind of approach not only enhances the freedom of the solution space for the presented stability criteria but is also subjected to the complexity in analysis. Recently, a projection approach was developed for linear uncertain time-delay systems in [17]. In addition to the simple stabilization, there have been various efforts in assigning certain performance criteria when designing a controller. One approach to this problem is the so-called guaranteed cost control first proposed in [3]. Its essential idea is to stabilize the systems while maintaining an adequate level of performance represented by a quadratic cost function. Some important results on guaranteed cost control have been presented (see, e.g., [6], [12], [20], [27], [28], and [32], where [12] and [32] studied delay-dependent guaranteed cost control problems for deterministic and stochastic T-S fuzzy systems with time delay, respectively). To the best of our knowledge, there exist a few previous delay-dependent guaranteed cost control results for stochastic fuzzy systems with multiple time delays in the literature, although many other results on multiple-time-delay systems have been obtained (see, e.g., [2], [4], and [31]). This motivates our research.

In this paper, we study the guaranteed cost control problem for stochastic fuzzy systems with multiple time delays and uncertain parameters. By employing a new Lyapunov–Krasovskii functional with an integral quadratic term and a new integral inequality technique, delay-dependent stability criteria are obtained such that the closed-loop stochastic fuzzy system is asymptotically stable in the mean-square sense with a guaranteed cost control performance. Then, a procedure is given to select a suitable controller that is optimal in the sense of minimizing

Manuscript received November 7, 2006; revised April 29, 2007. This work was supported in part by the National Natural Science Foundation of China under Grant 60534010, Grant 60572070, Grant 60728307, and Grant 60774048, by the Funds for Creative Research Groups of China under Grant 60521003, and by the Program for Cheung Kong Scholars and Innovative Research Team in University under Grant IRT0421. This paper was recommended by Associate Editor W. J. Wang.

H. Zhang is with the Institute of Electric Automation, Northeastern University, Shenyang 110004, China, and also with the Key Laboratory of Integrated Automation of Process Industry (Northeastern University), National Education Ministry, Shenyang 110004, China (e-mail: hgzhang@ieee.org).

Y. Wang is with the School of Information Science and Engineering, Northeastern University, Shenyang 110004, China, and also with the Key Laboratory of Integrated Automation of Process Industry (Northeastern University), National Education Ministry, Shenyang 110004, China (e-mail: yingchun_wang@163.com).

D. Liu is with the Department of Electrical and Computer Engineering, University of Illinois, Chicago, IL 60607 USA (e-mail: dliu@ece.uic.edu).

Digital Object Identifier 10.1109/TSMCB.2007.910532

the upper bound of the guaranteed cost function. All results are established in the form of linear matrix inequalities (LMIs) and can be easily solved [1]. One of the advantages is that neither system transformation nor relaxation matrices are required. In particular, some system transformation approaches may lead to conservatism in some cases, which has been pointed out in [11]. Another advantage is that the minimization of cost function can be directly solved by the LMI toolbox of Matlab, while the optimal control gain matrix can be obtained.

This paper is organized as follows. In Section II, the stochastic fuzzy system with multiple time delays and uncertain parameters is formulated. In Section III, the state feedback guaranteed cost control approach for uncertain stochastic fuzzy systems is developed. In Section IV, two simulation examples are provided to demonstrate the effectiveness of the present approach. In Section V, conclusions are given.

II. Problem Formulation and Preliminaries

Throughout this paper, for $h>0$, we let $C([-h,0];R^n)$ denote the family of continuous functions φ from $[-h,0]$ to R^n with the norm $\|\varphi\|=\sup_{-h\le\theta\le 0}|\varphi(\theta)|$, where $|\cdot|$ denotes the Euclidean norm in R^n. The notation $M>0$ $(M<0)$ is used to denote a positive (negative) definite symmetric matrix M. Moreover, let $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t>0},\mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t>0}$ that satisfies the usual conditions (i.e., the filtration contains all $\mathcal{P}$-null sets and is right continuous). Let $L^2_{\mathcal{F}_0}([-h,0];R^n)$ be the family of $\mathcal{F}_0$ measurable $C([-h,0];R^n)$-valued random variables $\zeta=\{\zeta(\theta):-h\le\theta\le 0\}$ such that $\sup_{-h\le\theta\le 0}\mathcal{E}\{|\zeta(\theta)|^2\}<\infty$, where $\mathcal{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$. We will use $*$ to denote the transposed elements in the symmetric positions of a matrix.

We first introduce two useful Lemmas, which will be used in the proof of our results.

Lemma 1 (cf. [23]): For matrices $A\in R^{n\times n}$, $P\in R^{n\times n}$, $M\in R^{n\times k}$, $N\in R^{l\times n}$, and $F\in R^{k\times l}$, with $P>0$, $F^TF\le I$, and a scalar $\varepsilon>0$, the following matrix inequalities hold:

1) $(MFN)^TP+PMFN\le\varepsilon PMM^TP+\varepsilon^{-1}N^TN$;
2) If $P-\varepsilon MM^T>0$, then $(A+MFN)^TP^{-1}(A+MFN)\le A^T(P-\varepsilon MM^T)^{-1}A+\varepsilon^{-1}N^TN$.

Lemma 2: For any constant positive definite symmetric matrix $W\in R^{m\times m}$, scalars $\beta>0$ and $\kappa>0$, and vector function $\bar{\upsilon}:[\beta-\kappa,\beta]\to R^{m\times 1}$, such that the integrations in the following are well defined, we have

$$\kappa\int_{\beta-\kappa}^{\beta}\bar{\upsilon}^T(s)W\bar{\upsilon}(s)ds\ge\left(\int_{\beta-\kappa}^{\beta}\bar{\upsilon}(s)ds\right)^TW\int_{\beta-\kappa}^{\beta}\bar{\upsilon}(s)ds.$$

The proof of Lemma 2 can be found in the Appendix.

Remark 1: Lemma 2 is similar to Lemma 1 in [10]. The only difference between them is that the lower limit of the integrations in the present case may be less than zero. When $\kappa=\beta$, it becomes the same as Lemma 1 in [10].

Now, we consider a class of uncertain stochastic fuzzy systems with multiple time delays, in which the ith rule is formulated in the following form:

Rule i :
IF $z_1(t)$ is $\mathcal{R}_{i1},\ldots,$ and $z_p(t)$ is $\mathcal{R}_{ip}$
THEN

$$\begin{aligned}dx(t)&=\sum_{k=0}^{m}(B_{ik}+\Delta B_{ik}(t))\,x(t-h_k)dt+D_iu(t)dt\\&\quad+\sum_{k=0}^{m}(C_{ik}+\Delta C_{ik}(t))\,x(t-h_k)dw(t)\\x(t)&=\zeta(t),\qquad t\in[-h,0]\end{aligned}\tag{1}$$

where $i=1,\ldots,r$; r is the number of fuzzy rules; $z_1(t),\ldots,z_p(t)$ are the premise variables; $\mathcal{R}_{ij}$ are the fuzzy sets, $j=1,\ldots,p$; $x(t)\in R^n$ is the state vector; $u(t)\in R^q$ is the control input; $h_0=0$; $h_k>0$, $k=1,\ldots,m$, denote the state delay; $h=\max\{h_k,k\in[1,m]\}$; $w(t)$ is a standard Brownian motion; and $\zeta(t)\in R^n$ is a continuous initial function or random variable. It is assumed that the premise variables do not depend on the input noise $w(t)$ explicitly. B_{ik}, C_{ik}, and D_i are the known matrices with compatible dimensions. The uncertain matrix functions $\Delta B_{ik}(t)$ and $\Delta C_{ik}(t)$ satisfy the following condition:

$$[\Delta B_{ik}(t)\quad\Delta C_{ik}(t)]=M_iF_i(t)\,[N_{1ik}\quad N_{2ik}]\tag{2}$$

where $M_i\in R^{n\times f}$, $N_{1ik}\in R^{f\times n}$, and $N_{2ik}\in R^{f\times n}$, $k=0,\ldots,m$, are known constant matrices. $F_i(t)$ is an unknown matrix function with Lebesgue measurable elements and satisfies $F_i^T(t)F_i(t)\le I\in R^{f\times f}$, where I is the identity matrix.

The uncertain stochastic fuzzy system (1) is inferred as follows:

$$dx(t)=\sum_{k=0}^{m}B_k(\delta)x(t-h_k)dt+D(\delta)u(t)dt+\sum_{k=0}^{m}C_k(\delta)x(t-h_k)dw(t)\tag{3}$$

where $B_k(\delta)=\sum_{i=1}^r\delta_i(z(t))(B_{ik}+\Delta B_{ik}(t))$, $C_k(\delta)=\sum_{i=1}^r\delta_i(z(t))(C_{ik}+\Delta C_{ik}(t))$, $D(\delta)=\sum_{i=1}^r\delta_i(z(t))D_i$, $\delta_i(z(t))=\sigma_i(z(t))/\sum_{i=1}^r\sigma_i(z(t))$, $\sigma_i(z(t))=\prod_{l=1}^p\mathcal{R}_{il}(z_l(t))$, and $\mathcal{R}_{il}(z_l(t))$ is the membership function of $z_l(t)$ in $\mathcal{R}_{il}$, $l=1,\ldots,p$.

Assume that $\sigma_i(z(t))\ge 0$ and $\sum_{i=1}^r\sigma_i(z(t))>0$ for all t. Therefore, we get $\delta_i(z(t))\ge 0$ for $i=1,\ldots,r$ and $\sum_{i=1}^r\delta_i(z(t))=1$.

We use the controller structure incorporating a set of fuzzy rules expressed in the form

$$\begin{aligned}\text{Rule } i:\quad&\text{IF } z_1(t) \text{ is } \mathcal{R}_{i1},\ldots,\text{and } z_p(t) \text{ is } \mathcal{R}_{ip}\\&\text{THEN } u(t)=K_ix(t).\end{aligned}\tag{4}$$

Hence, the inferred fuzzy controller is given by

$$u(t)=\sum_{i=1}^{r}\delta_i\left(z(t)\right)K_ix(t)\tag{5}$$

where K_i is the local control gain matrix to be determined.

Substituting (5) into (3), we have the following closed-loop form of the stochastic fuzzy system:

$$dx(t) = \sum_{k=0}^{m} (B_{bk}(\delta) + \Delta B_{bk}(\delta))\, x(t-h_k)dt + \sum_{k=0}^{m} (C_{bk}(\delta) + \Delta C_{bk}(\delta))\, x(t-h_k)dw(t) \quad (6)$$

where the expressions for $B_{bk}(\delta)$, $\Delta B_{bk}(\delta)$, $C_{bk}(\delta)$, and $\Delta C_{bk}(\delta)$ are shown as

$$B_{bk}(\delta) = \begin{cases} \sum_{i=1}^{r}\sum_{j=1}^{r} \delta_i(z(t)) \\ \quad \times\delta_j(z(t))(B_{ik}+D_iK_j), & \text{for } k=0 \\ \sum_{i=1}^{r}\sum_{j=1}^{r} \delta_i(z(t))\,\delta_j(z(t)) \\ \quad \times B_{ik} = \sum_{i=1}^{r}\delta_i(z(t))B_{ik}, & \text{for } k=1,\ldots,m \end{cases}$$

$$\Delta B_{bk}(\delta) = \sum_{i=1}^{r}\sum_{j=1}^{r} \delta_i(z(t))\,\delta_j(z(t))\,\Delta B_{ik}(t) = \sum_{i=1}^{r}\delta_i(z(t))\,\Delta B_{ik}(t)$$

$$C_{bk}(\delta) = \sum_{i=1}^{r}\sum_{j=1}^{r} \delta_i(z(t))\,\delta_j(z(t))\,C_{ik} = \sum_{i=1}^{r}\delta_i(z(t))\,C_{ik}$$

$$\Delta C_{bk}(\delta) = \sum_{i=1}^{r}\sum_{j=1}^{r} \delta_i(z(t))\,\delta_j(z(t))\,\Delta C_{ik}(t) = \sum_{i=1}^{r}\delta_i(z(t))\,\Delta C_{ik}(t)$$

The stability of stochastic fuzzy system (3) is defined as follows.

Definition 1: For system (3) with $u(t) = 0$, the trivial solution is asymptotically stable in the mean-square sense for every $\zeta \in L^2_{\mathcal{F}_0}([-h,0];R^n)$ if

$$\lim_{t\to\infty} \mathcal{E}\,|x(t,\zeta)|^2 = 0.$$

Given positive definite symmetric matrices Ξ and Ψ, we shall consider the cost function

$$J = \mathcal{E}\left\{ \int_0^{\infty} \left[x^T(t)\Xi x(t) + u^T(t)\Psi u(t)\right] dt \right\}. \quad (7)$$

Associated with the cost function, the guaranteed cost controller is defined as follows.

Definition 2: Consider system (3). If there exist a control law $u^*(t)$ and a scalar $J^* > 0$ such that the resulting closed-loop system is asymptotically stable in the mean-square sense and the value of cost function (7) satisfies $J \le J^*$, then J^* is said to be a guaranteed cost, and $u^*(t)$ is said to be a guaranteed cost control law for system (3).

Our objective is to develop a delay-dependent stabilization approach, which provides the state feedback control gain matrix as well as a positive scalar J^* such that the closed-loop system is asymptotically stable in the mean-square sense and the value of cost function (7) satisfies $J \le J^*$.

III. Main Result

In this section, we develop our main results for the stochastic fuzzy system (6). We now state and prove our first result.

Theorem 1: Given $h_k > 0$, $k = 1,\ldots,m$, the closed-loop stochastic fuzzy system (6) is asymptotically stable in the mean-square sense, if there exist matrices $X > 0$, $\hat{R} > 0$, $\hat{Q} > 0$, and $\hat{K}_i$ $(i = 1,\ldots,r)$ with compatible dimensions and scalars $\varepsilon_{i1} > 0$ and $\varepsilon_{i2} > 0$, such that the LMIs (8), shown at the bottom of the page, hold for $1 \le i \le j \le r$ where

$$\begin{aligned}
\Pi_{1,ij} &= \Big(B_{i0}X + XB_{i0}^T + B_{j0}X + XB_{j0}^T + D_j\hat{K}_i + D_i\hat{K}_j \\
&\quad + \hat{K}_i^TD_j^T + \hat{K}_j^TD_i^T + \varepsilon_{i1}M_iM_i^T + \varepsilon_{j1}M_jM_j^T\Big) \in R^{n\times n}\\
\Pi_{2,ij}^k &= (B_{ik}X + B_{jk}X) \in R^{n\times n}\\
\Pi_{3,ij}^k &= (-4X + 2\hat{Q}) \in R^{n\times n}\\
\Pi_{4,ij}^k &= 2h_kX \in R^{n\times n}\\
\Pi_{5,ij}^k &= -2h_kX \in R^{n\times n}\\
\Pi_{6,ij}^k &= -4h_kX + 2h_k\hat{R} \in R^{n\times n}\\
\Pi_{7,ij} &= \begin{bmatrix} XC_{i0}^T & XC_{j0}^T & XN_{2i0}^T & XN_{2j0}^T \end{bmatrix} \in R^{n\times(2n+2f)}\\
\Pi_{7,ij}^k &= \begin{bmatrix} XC_{ik}^T & XC_{jk}^T & XN_{2ik}^T & XN_{2jk}^T \end{bmatrix} \in R^{n\times(2n+2f)}
\end{aligned}$$

$$\begin{bmatrix}
\Pi_{1,ij} & \Pi_{2,ij}^1 & \ldots & \Pi_{2,ij}^m & \Pi_{4,ij}^1 & \ldots & \Pi_{4,ij}^m & \Pi_{7,ij} & \Pi_{9,ij} & \Pi_{11,ij}\\
* & \Pi_{3,ij}^1 & 0 & 0 & \Pi_{5,ij}^1 & 0 & 0 & \Pi_{7,ij}^1 & \Pi_{9,ij}^1 & 0\\
* & * & \ddots & 0 & 0 & \ddots & 0 & \vdots & \vdots & 0\\
* & * & * & \Pi_{3,ij}^m & 0 & 0 & \Pi_{5,ij}^m & \Pi_{7,ij}^m & \Pi_{9,ij}^m & 0\\
* & * & * & * & \Pi_{6,ij}^1 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & \ddots & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & \Pi_{6,ij}^m & 0 & 0 & 0\\
* & * & * & * & * & * & * & \Pi_{8,ij} & 0 & 0\\
* & * & * & * & * & * & * & * & \Pi_{10,ij} & 0\\
* & * & * & * & * & * & * & * & * & \Pi_{12,ij}
\end{bmatrix} < 0 \quad (8)$$

$$\begin{aligned}
\Pi_{8,ij} &= -\mathrm{diag}\big(X - \varepsilon_{i2} M_i M_i^T, X - \varepsilon_{j2} M_j M_j^T, \varepsilon_{i2} I, \varepsilon_{j2} I\big) \\
&\quad \in R^{(2n+2f)\times(2n+2f)} \\
\Pi_{9,ij} &= \begin{bmatrix} X N_{1i0}^T & X N_{1j0}^T \end{bmatrix} \in R^{n\times 2f} \\
\Pi_{9,ij}^k &= \begin{bmatrix} X N_{1ik}^T & X N_{1jk}^T \end{bmatrix} \in R^{n\times 2f} \\
\Pi_{10,ij} &= -\mathrm{diag}(\varepsilon_{i1} I, \varepsilon_{j1} I) \in R^{2f\times 2f} \\
\Pi_{11,ij} &= \begin{bmatrix} 2h_d X & 2mX & 2X & \hat{K}_i^T & \hat{K}_j^T \end{bmatrix} \in R^{n\times(3n+2q)} \\
\Pi_{12,ij} &= -\mathrm{diag}(2h_d \hat{R}, 2m\hat{Q}, 2\Xi^{-1}, \Psi^{-1}, \Psi^{-1}) \\
&\quad \in R^{(3n+2q)\times(3n+2q)} \\
k &= 1,\ldots,m; \quad h_d = \sum_{k=1}^{m} h_k.
\end{aligned}$$

Moreover, the control gain matrix can be chosen as $K_i = \hat{K}_i X^{-1}$, and the guaranteed cost bound is determined as

$$\begin{aligned}
J^* &= \mathcal{E}\left\{x^T(0) X^{-1} x(0)\right\} \\
&\quad + \mathcal{E}\left\{\sum_{k=1}^{m} \int_{-h_k}^{0} x^T(\tau)\hat{Q}^{-1} x(\tau) d\tau\right\} \\
&\quad + \mathcal{E}\left\{\sum_{k=1}^{m} \int_{-h_k}^{0}\int_{\beta}^{0} x^T(\tau)\hat{R}^{-1} x(\tau) d\tau d\beta\right\} \\
&\quad + \mathcal{E}\left\{\sum_{k=1}^{m} \left(\int_{-h_k}^{0} x(\tau) d\tau\right)^T X^{-1} \int_{-h_k}^{0} x(\tau) d\tau\right\}. \quad (9)
\end{aligned}$$

Proof: Define the following Lyapunov–Krasovskii functional:

$$\begin{aligned}
V(x,t) &= x^T(t) P x(t) + \sum_{k=1}^{m} \int_{-h_k}^{0}\int_{t+\beta}^{t} x^T(\tau) R x(\tau) d\tau d\beta \\
&\quad + \sum_{k=1}^{m} \int_{t-h_k}^{t} x^T(\tau) Q x(\tau) d\tau \\
&\quad + \sum_{k=1}^{m} \left(\int_{t-h_k}^{t} x(\tau) d\tau\right)^T P \int_{t-h_k}^{t} x(\tau) d\tau \qquad (10)
\end{aligned}$$

where $P = X^{-1}$, $R = \hat{R}^{-1}$, and $Q = \hat{Q}^{-1}$. By the Itô formula [16], we obtain

$$\begin{aligned}
dV(x,t) &= \mathcal{L}V(x,t)dt + 2x^T(t) P \sum_{k=0}^{m} (C_{bk}(\delta) \\
&\quad + \Delta C_{bk}(\delta))\, x(t-h_k) dw(t) \quad (11)
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{L}V(x,t) &= 2x^T(t) P \sum_{k=0}^{m} (B_{bk}(\delta) + \Delta B_{bk}(\delta))\, x(t-h_k) \\
&\quad + \left(\sum_{k=0}^{m} (C_{bk}(\delta) + \Delta C_{bk}(\delta))\, x(t-h_k)\right)^T P \\
&\quad \times \sum_{k=0}^{m} (C_{bk}(\delta) + \Delta C_{bk}(\delta))\, x(t-h_k) \\
&\quad + \sum_{k=1}^{m} \left(h_k x^T(t) R x(t) - \int_{t-h_k}^{t} x^T(\tau) R x(\tau) d\tau\right) \\
&\quad + \sum_{k=1}^{m} \left(x^T(t) Q x(t) - x^T(t-h_k) Q x(t-h_k)\right) \\
&\quad + \sum_{k=1}^{m} \left(2\left(x(t) - x(t-h_k)\right)^T P \int_{t-h_k}^{t} x(\tau) d\tau\right). \qquad (12)
\end{aligned}$$

Using Lemma 1 and considering the uncertain parameters (2), we obtain

$$\begin{aligned}
2x^T(t) P \sum_{k=0}^{m} \Delta B_{bk}(\delta) x(t-h_k) &= \sum_{i=1}^{r} \delta_i(z(t)) \\
&\quad \times 2x^T(t) P M_i F_i(t) \bar{N}_i \psi(t) \\
&\le \sum_{i=1}^{r} \delta_i(z(t)) \big(\varepsilon_{i1} x^T(t) P M_i M_i^T P x(t) \\
&\quad + \varepsilon_{i1}^{-1} \psi^T(t) \bar{N}_i^T \bar{N}_i \psi(t)\big) \qquad (13)
\end{aligned}$$

where

$$\begin{aligned}
\bar{N}_i &= [N_{1i0}\ N_{1i1}\ \cdots\ N_{1ik}\ \cdots\ N_{1im}] \\
\psi(t) &= \left[x^T(t)\ x^T(t-h_1)\ \cdots\ x^T(t-h_k)\ \cdots\ x^T(t-h_m)\right]^T.
\end{aligned}$$

Using Lemma 1, we can also obtain

$$\begin{aligned}
&\left(\sum_{k=0}^{m} (C_{bk}(\delta) + \Delta C_{bk}(\delta))\, x(t-h_k)\right)^T P \\
&\times \sum_{k=0}^{m} (C_{bk}(\delta) + \Delta C_{bk}(\delta))\, x(t-h_k) \\
&\le \frac{1}{2} \sum_{i=1}^{r}\sum_{j=1}^{r} \delta_i(z(t))\, \delta_j(z(t))\, \psi^T(t) \\
&\quad \times \Big((W_i + \Delta W_i(t))^T P (W_i + \Delta W_i(t)) \\
&\quad + (W_j + \Delta W_j(t))^T P (W_j + \Delta W_j(t))\Big) \psi(t) \\
&= \sum_{i=1}^{r} \delta_i(z(t))\, \psi^T(t) \\
&\quad \times (W_i + \Delta W_i(t))^T P (W_i + \Delta W_i(t))\, \psi(t) \\
&\le \sum_{i=1}^{r} \delta_i(z(t))\, \psi^T(t) \\
&\quad \times \left(W_i^T \left(P^{-1} - \varepsilon_{i2} M_i M_i^T\right)^{-1} W_i + \varepsilon_{i2}^{-1} \bar{\mathcal{N}}_i^T \bar{\mathcal{N}}_i\right) \psi(t) \qquad (14)
\end{aligned}$$

where

$$\begin{aligned}
W_i &= [C_{i0}\ C_{i1}\ \cdots\ C_{ik}\ \cdots\ C_{im}] \\
\Delta W_i(t) &= [\Delta C_{i0}(t)\ \Delta C_{i1}(t)\ \cdots\ \Delta C_{ik}(t)\ \cdots\ \Delta C_{im}(t)] \\
&= M_i F_i(t) \bar{\mathcal{N}}_i \\
\bar{\mathcal{N}}_i &= [N_{2i0}\ N_{2i1}\ \cdots\ N_{2ik}\ \cdots\ N_{2im}].
\end{aligned}$$

Using Lemma 2, we have

$$-\int_{t-h_k}^{t} x^T(\tau)Rx(\tau)d\tau \le -h_k^{-1}\left(\int_{t-h_k}^{t} x(\tau)d\tau\right)^T R\int_{t-h_k}^{t} x(\tau)d\tau. \tag{15}$$

Substituting (13)–(15) into (12), we have (16), shown at the bottom of the page, where

$$\bar{\xi}^T(t) = \left[x^T(t) \quad x^T(t-h_1) \quad \cdots \quad x^T(t-h_m) \quad \left(\int_{t-h_1}^{t} x(\tau)d\tau\right)^T \quad \cdots \quad \left(\int_{t-h_m}^{t} x(\tau)d\tau\right)^T\right]$$

and the rest of the notation is expressed in

$$Z_{ij}+Z_{ji} = \begin{bmatrix} Z_{1,ij} & Z_{2,ij}^1 & Z_{2,ij}^2 & \cdots & Z_{2,ij}^m & Z_{4,ij}^1 & Z_{4,ij}^2 & \cdots & Z_{4,ij}^m \\ * & Z_{3,ij}^{1,1} & Z_{3,ij}^{1,2} & \cdots & Z_{3,ij}^{1,m} & Z_{5,ij}^1 & 0 & 0 & 0 \\ * & * & Z_{3,ij}^{2,2} & \cdots & Z_{3,ij}^{2,m} & 0 & Z_{5,ij}^2 & 0 & 0 \\ * & * & * & \ddots & \vdots & 0 & 0 & \ddots & 0 \\ * & * & * & * & Z_{3,ij}^{m,m} & 0 & 0 & 0 & Z_{5,ij}^m \\ * & * & * & * & * & Z_{6,ij}^1 & 0 & 0 & 0 \\ * & * & * & * & * & * & Z_{6,ij}^2 & 0 & 0 \\ * & * & * & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & * & * & * & Z_{6,ij}^m \end{bmatrix} \tag{17}$$

and $Z_{ii} = (1/2)(Z_{ij} + Z_{ji})$, for $i = j$, with

$$\begin{aligned} Z_{1,ij} =& P(B_{i0} + D_iK_j + B_{j0} + D_jK_i) \\ &+ (B_{i0} + D_iK_j + B_{j0} + D_jK_i)^T P \\ &+ \varepsilon_{i1}PM_iM_i^TP + \varepsilon_{j1}PM_jM_j^TP \\ &+ \varepsilon_{i1}^{-1}N_{1i0}^TN_{1i0} + \varepsilon_{j1}^{-1}N_{1j0}^TN_{1j0} \\ &+ C_{i0}^T\left(P^{-1} - \varepsilon_{i2}M_iM_i^T\right)^{-1}C_{i0} \\ &+ C_{j0}^T\left(P^{-1} - \varepsilon_{j2}M_jM_j^T\right)^{-1}C_{j0} \\ &+ \varepsilon_{i2}^{-1}N_{2i0}^TN_{2i0} + \varepsilon_{j2}^{-1}N_{2j0}^TN_{2j0} + 2h_dR + 2mQ \\ &+ 2\Xi + K_i^T\Psi K_i + K_j^T\Psi K_j \\ Z_{2,ij}^k =& P(B_{ik} + B_{jk}) + \varepsilon_{i1}^{-1}N_{1i0}^TN_{1ik} + \varepsilon_{j1}^{-1}N_{1j0}^TN_{1jk} \\ &+ C_{i0}^T\left(P^{-1} - \varepsilon_{i2}M_iM_i^T\right)^{-1}C_{ik} \\ &+ C_{j0}^T\left(P^{-1} - \varepsilon_{j2}M_jM_j^T\right)^{-1}C_{jk} \\ &+ \varepsilon_{i2}^{-1}N_{2i0}^TN_{2ik} + \varepsilon_{j2}^{-1}N_{2j0}^TN_{2jk} \\ Z_{3,ij}^{k,k} =& -2Q + \varepsilon_{i1}^{-1}N_{1ik}^TN_{1ik} + \varepsilon_{j1}^{-1}N_{1jk}^TN_{1jk} \\ &+ C_{ik}^T\left(P^{-1} - \varepsilon_{i2}M_iM_i^T\right)^{-1}C_{ik} \\ &+ C_{jk}^T\left(P^{-1} - \varepsilon_{j2}M_jM_j^T\right)^{-1}C_{jk} \\ &+ \varepsilon_{i2}^{-1}N_{2ik}^TN_{2ik} + \varepsilon_{j2}^{-1}N_{2jk}^TN_{2jk} \\ Z_{3,ij}^{k,l} =& \varepsilon_{i1}^{-1}N_{1ik}^TN_{1il} + \varepsilon_{j1}^{-1}N_{1jk}^TN_{1jl} \\ &+ C_{ik}^T\left(P^{-1} - \varepsilon_{i2}M_iM_i^T\right)^{-1}C_{il} \\ &+ C_{jk}^T\left(P^{-1} - \varepsilon_{j2}M_jM_j^T\right)^{-1}C_{jl} \\ &+ \varepsilon_{i2}^{-1}N_{2ik}^TN_{2il} + \varepsilon_{j2}^{-1}N_{2jk}^TN_{2jl} \qquad (l > k) \\ Z_{4,ij}^k =& 2P \qquad Z_{5,ij}^k = -2P \\ Z_{6,ij}^k =& -2h_k^{-1}R, \qquad k = 1, \ldots, m. \end{aligned}$$

$$\begin{aligned} \mathcal{L}V(x,t) \le& 2x^T(t)P\sum_{k=0}^{m} B_{bk}(\delta)x(t-h_k) + \sum_{i=1}^{r}\delta_i\left(z(t)\right)\left(\varepsilon_{i1}x^T(t)PM_iM_i^TPx(t) + \varepsilon_{i1}^{-1}\psi^T(t)\bar{N}_i^T\bar{N}_i\psi(t)\right) \\ &+ \sum_{i=1}^{r}\delta_i\left(z(t)\right)\psi^T(t)\left(W_i^T(P^{-1} - \varepsilon_{i2}M_iM_i^T)^{-1}W_i + \varepsilon_{i2}^{-1}\bar{\mathcal{N}}_i^T\bar{\mathcal{N}}_i\right)\psi(t) \\ &+ \sum_{k=1}^{m}\left(h_kx^T(t)Rx(t) - h_k^{-1}\left(\int_{t-h_k}^{t} x(\tau)d\tau\right)^T R\int_{t-h_k}^{t} x(\tau)d\tau\right) + \sum_{k=1}^{m}\left(x^T(t)Qx(t) - x^T(t-h_k)Qx(t-h_k)\right) \\ &+ \sum_{k=1}^{m}\left(2\left(x(t) - x(t-h_k)\right)^TP\int_{t-h_k}^{t} x(\tau)d\tau\right) + x^T(t)\Xi x(t) + u^T(t)\Psi u(t) - x^T(t)\Xi x(t) - u^T(t)\Psi u(t) \\ \le& \sum_{i=1}^{r}\sum_{j=1}^{r}\delta_i\left(z(t)\right)\delta_j\left(z(t)\right)\bar{\xi}^T(t)Z_{ij}\bar{\xi}(t) - x^T(t)\Xi x(t) - u^T(t)\Psi u(t) \\ =& \sum_{i=1}^{r}\sum_{j>i}^{r}\delta_i\left(z(t)\right)\delta_j\left(z(t)\right)\bar{\xi}^T(t)(Z_{ij}+Z_{ji})\bar{\xi}(t) + \sum_{i=1}^{r}\delta_i^2\left(z(t)\right)\bar{\xi}^T(t)Z_{ii}\bar{\xi}(t) - x^T(t)\Xi x(t) - u^T(t)\Psi u(t) \end{aligned} \tag{16}$$

Note that the following result has been used in (16):

$$
\begin{aligned}
u^T(t)\Psi u(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\delta_i\left(z(t)\right)\delta_j\left(z(t)\right)x^T(t)K_i^T\Psi K_j x(t)\\
&\le \frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r}\delta_i\left(z(t)\right)\delta_j\left(z(t)\right)x^T(t)\\
&\quad\times\left(K_i^T\Psi K_i + K_j^T\Psi K_j\right)x(t)\\
&= \sum_{i=1}^{r}\delta_i\left(z(t)\right)x^T(t)K_i^T\Psi K_i x(t). \qquad (18)
\end{aligned}
$$

If $Z_{ij}+Z_{ji}<0$ holds for all $1\le i\le j\le r$, then $\mathcal{L}V(x,t)<0$ for every $\bar{\xi}(t)\neq 0$.

Because $X=P^{-1}$, $\hat{Q}=Q^{-1}$, and $\hat{R}=R^{-1}$, we can let $\hat{K}_i=K_iX$. Pre- and postmultiplying $\mathrm{diag}(P^{-1}, \underbrace{P^{-1},\ldots,P^{-1}}_{m}, h_1P^{-1},\ldots,h_mP^{-1})$ to the left-hand side of inequality $Z_{ij}+Z_{ji}<0$ [cf. (17)] and using the Schur complement, we obtain (19), shown at the bottom of the page, where $\bar{\Pi}^k_{3,ij}=-2XQX$, $\bar{\Pi}^k_{6,ij}=-2h_kXRX$, $k=1,\ldots,m$, and other notations are defined as in (8).

The inequality (19) is not a solvable LMI because of the nonlinear terms XQX and XRX in $\bar{\Pi}^k_{3,ij}$ and $\bar{\Pi}^k_{6,ij}$, respectively. Because X and Q are positive definite symmetric matrices, we have

$$(X-Q^{-1})^TQ(X-Q^{-1})=(X-Q^{-1})Q(X-Q^{-1})\ge 0$$

then

$$-XQX\le -2X+Q^{-1}. \qquad (20)$$

Similarly we have

$$-XRX\le -2X+R^{-1}. \qquad (21)$$

Because $\hat{R}=R^{-1}$ and $\hat{Q}=Q^{-1}$, from (19)–(21), we obtain (8), which guarantees $Z_{ij}+Z_{ji}<0$ $(1\le i\le j\le r)$. Moreover, from (16), we have

$$\mathcal{L}V(x,t)\le -x^T(t)\Xi x(t)-u^T(t)\Psi u(t)<0. \qquad (22)$$

Therefore, system (6) is asymptotically stable in the mean-square sense with the control gain matrix $K_i=\hat{K}_iX^{-1}$.

Integrating inequality (11) from 0 to $T>0$, taking the mathematical expectation, and considering inequality (22), we obtain

$$
\begin{aligned}
&\mathcal{E}\left\{V\left(x(T),T\right)\right\}-\mathcal{E}\left\{V\left(x(0),0\right)\right\}\\
&\quad=\mathcal{E}\left\{x^T(T)Px(T)\right\}+\mathcal{E}\left\{\sum_{k=1}^{m}\int_{T-h_k}^{T}x^T(\tau)Qx(\tau)d\tau\right\}\\
&\qquad+\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0}\int_{T+\beta}^{T}x^T(\tau)Rx(\tau)d\tau d\beta\right\}\\
&\qquad+\mathcal{E}\left\{\sum_{k=1}^{m}\left(\int_{T-h_k}^{T}x(\tau)d\tau\right)^T P\int_{T-h_k}^{T}x(\tau)d\tau\right\}\\
&\qquad-\mathcal{E}\left\{x^T(0)Px(0)\right\}-\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0}x^T(\tau)Qx(\tau)d\tau\right\}\\
&\qquad-\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0}\int_{\beta}^{0}x^T(\tau)Rx(\tau)d\tau d\beta\right\}\\
&\qquad-\mathcal{E}\left\{\sum_{k=1}^{m}\left(\int_{-h_k}^{0}x(\tau)d\tau\right)^T P\int_{-h_k}^{0}x(\tau)d\tau\right\}\\
&\quad=\mathcal{E}\left\{\int_{0}^{T}\mathcal{L}V(x,t)dt\right\}\\
&\quad\le -\mathcal{E}\left\{\int_{0}^{T}\left(x^T(\tau)\Xi x(\tau)+u^T(\tau)\Psi u(\tau)\right)d\tau\right\}. \qquad (23)
\end{aligned}
$$

$$
\begin{bmatrix}
\Pi_{1,ij} & \Pi^1_{2,ij} & \cdots & \Pi^m_{2,ij} & \Pi^1_{4,ij} & \cdots & \Pi^m_{4,ij} & \Pi_{7,ij} & \Pi_{9,ij} & \Pi_{11,ij}\\
* & \bar{\Pi}^1_{3,ij} & 0 & 0 & \Pi^1_{5,ij} & 0 & 0 & \Pi^1_{7,ij} & \Pi^1_{9,ij} & 0\\
* & * & \ddots & 0 & 0 & \ddots & 0 & \vdots & \vdots & 0\\
* & * & * & \bar{\Pi}^m_{3,ij} & 0 & 0 & \Pi^m_{5,ij} & \Pi^m_{7,ij} & \Pi^m_{9,ij} & 0\\
* & * & * & * & \bar{\Pi}^1_{6,ij} & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & \ddots & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & \bar{\Pi}^m_{6,ij} & 0 & 0 & 0\\
* & * & * & * & * & * & * & \Pi_{8,ij} & 0 & 0\\
* & * & * & * & * & * & * & * & \Pi_{10,ij} & 0\\
* & * & * & * & * & * & * & * & * & \Pi_{12,ij}
\end{bmatrix} < 0 \qquad (19)
$$

Because system (6) is asymptotically stable in the mean-square sense, when $T \to \infty$, we have

$$\mathcal{E}\left\{x^T(T)Px(T)\right\} \to 0$$
$$\mathcal{E}\left\{\sum_{k=1}^{m}\int_{T-h_k}^{T} x^T(\tau)Qx(\tau)d\tau\right\} \to 0$$
$$\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0}\int_{T+\beta}^{T} x^T(\tau)Rx(\tau)d\tau d\beta\right\} \to 0$$
$$\mathcal{E}\left\{\sum_{k=1}^{m}\left(\int_{T-h_k}^{T} x(\tau)d\tau\right)^T P\int_{T-h_k}^{T} x(\tau)d\tau\right\} \to 0.$$

Hence, we have

$$\begin{aligned}\mathcal{E}&\left\{\int_0^\infty \left(x^T(\tau)\Xi x(\tau)+u^T(\tau)\Psi u(\tau)\right)d\tau\right\}\\ &\le \mathcal{E}\left\{x^T(0)Px(0)\right\}+\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0} x^T(\tau)Qx(\tau)d\tau\right\}\\ &\quad+\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0}\int_{\beta}^{0} x^T(\tau)Rx(\tau)d\tau d\beta\right\}\\ &\quad+\mathcal{E}\left\{\sum_{k=1}^{m}\left(\int_{-h_k}^{0} x(\tau)d\tau\right)^T P\int_{-h_k}^{0} x(\tau)d\tau\right\}\end{aligned} \tag{24}$$

that is

$$\begin{aligned}J=&\,\mathcal{E}\left\{\int_0^\infty \left(x^T(\tau)\Xi x(\tau)+u^T(\tau)\Psi u(\tau)\right)d\tau\right\}\\ \le&\,\mathcal{E}\left\{x^T(0)X^{-1}x(0)\right\}\\ &+\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0} x^T(\tau)\hat{Q}^{-1}x(\tau)d\tau\right\}\\ &+\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0}\int_{\beta}^{0} x^T(\tau)\hat{R}^{-1}x(\tau)d\tau d\beta\right\}\\ &+\mathcal{E}\left\{\sum_{k=1}^{m}\left(\int_{-h_k}^{0} x(\tau)d\tau\right)^T X^{-1}\int_{-h_k}^{0} x(\tau)d\tau\right\}\\ =&\,J^*.\end{aligned} \tag{25}$$

This completes the proof. ■

Note that the guaranteed cost bound in Theorem 1 depends on the choice of guaranteed cost controller. The guaranteed cost controller that minimizes the guaranteed cost is called an optimal guaranteed cost controller in [28]. Based on Theorem 1, the design problem of the optimal guaranteed cost controller is formulated as follows.

Theorem 2: Consider the stochastic fuzzy system (6) with cost function (7). If the following optimization problem

$$\min\left\{\mathrm{tr}(\Gamma_0)+\mathrm{tr}(\Gamma_1)+\mathrm{tr}(\Gamma_2)+\mathrm{tr}(\Gamma_3)\right\}$$
$$\text{s.t.}\begin{cases}\text{(i)} & \text{inequality (8)}\\ \text{(ii)} & \begin{bmatrix}-\Gamma_0 & \mathcal{Z}_0^T\\ \mathcal{Z}_0 & -X\end{bmatrix}<0\\ \text{(iii)} & \begin{bmatrix}-\Gamma_1 & \mathcal{Z}_1^T\\ \mathcal{Z}_1 & -\hat{R}\end{bmatrix}<0\\ \text{(iv)} & \begin{bmatrix}-\Gamma_2 & \mathcal{Z}_2^T\\ \mathcal{Z}_2 & -\hat{Q}\end{bmatrix}<0\\ \text{(v)} & \begin{bmatrix}-\Gamma_3 & \mathcal{Z}_3^T\\ \mathcal{Z}_3 & -X\end{bmatrix}<0\end{cases} \tag{26}$$

has a solution set $\Theta=(\varepsilon_{i1},\varepsilon_{i2},X,\hat{R},\hat{Q},\hat{K}_i,\Gamma_0,\Gamma_1,\Gamma_2,\Gamma_3,1\le i\le j\le r)$, where $\mathrm{tr}(\cdot)$ denotes the trace of a matrix, then controller (5) is an optimal guaranteed cost controller, which ensures the minimization of the guaranteed cost bound (9) for system (6), where

$$\mathcal{Z}_0\mathcal{Z}_0^T=\mathcal{E}\left\{x(0)x^T(0)\right\}$$
$$\mathcal{Z}_1\mathcal{Z}_1^T=\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0}\int_{\beta}^{0} x(\tau)x^T(\tau)d\tau d\beta\right\}$$
$$\mathcal{Z}_2\mathcal{Z}_2^T=\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0} x(\tau)x^T(\tau)d\tau\right\}$$
$$\mathcal{Z}_3\mathcal{Z}_3^T=\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0} x(\tau)d\tau\left(\int_{-h_k}^{0} x(\tau)d\tau\right)^T\right\}.$$

Proof: By Theorem 1, (i) in (26) is clear.

By the Schur complement, it follows that (ii), (iii), (iv), and (v) in (26) are equivalent to $\mathcal{Z}_0^TX^{-1}\mathcal{Z}_0<\Gamma_0$, $\mathcal{Z}_1^T\hat{R}^{-1}\mathcal{Z}_1<\Gamma_1$, $\mathcal{Z}_2^T\hat{Q}^{-1}\mathcal{Z}_2<\Gamma_2$, and $\mathcal{Z}_3^TX^{-1}\mathcal{Z}_3<\Gamma_3$, respectively. On the other hand

$$\begin{aligned}\mathcal{E}\left\{x^T(0)X^{-1}x(0)\right\}&=\mathrm{tr}\left(\mathcal{E}\left\{x^T(0)X^{-1}x(0)\right\}\right)\\ &=\mathrm{tr}\left(X^{-1}\mathcal{E}\left\{x(0)x^T(0)\right\}\right)\\ &=\mathrm{tr}\left(X^{-1}\mathcal{Z}_0\mathcal{Z}_0^T\right)\\ &=\mathrm{tr}\left(\mathcal{Z}_0^TX^{-1}\mathcal{Z}_0\right)\\ &<\mathrm{tr}(\Gamma_0)\end{aligned} \tag{27}$$

and similarly

$$\begin{aligned}
&\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0}\int_{\beta}^{0} x^T(\tau)\hat{R}^{-1}x(\tau)d\tau d\beta\right\}\\
&\quad=\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0}\int_{\beta}^{0}\operatorname{tr}\left(x^T(\tau)\hat{R}^{-1}x(\tau)\right)d\tau d\beta\right\}\\
&\quad=\operatorname{tr}\left(\hat{R}^{-1}\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0}\int_{\beta}^{0} x(\tau)x^T(\tau)d\tau d\beta\right\}\right)\\
&\quad=\operatorname{tr}\left(\hat{R}^{-1}\mathcal{Z}_1\mathcal{Z}_1^T\right)<\operatorname{tr}(\Gamma_1) \qquad (28)\\
&\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0} x^T(\tau)\hat{Q}^{-1}x(\tau)d\tau\right\}\\
&\quad=\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0}\operatorname{tr}\left(x^T(\tau)\hat{Q}^{-1}x(\tau)\right)d\tau\right\}\\
&\quad=\operatorname{tr}\left(\hat{Q}^{-1}\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0} x(\tau)x^T(\tau)d\tau\right\}\right)\\
&\quad=\operatorname{tr}\left(\hat{Q}^{-1}\mathcal{Z}_2\mathcal{Z}_2^T\right)<\operatorname{tr}(\Gamma_2) \qquad (29)\\
&\mathcal{E}\left\{\sum_{k=1}^{m}\left(\int_{-h_k}^{0} x(\tau)d\tau\right)^T X^{-1}\int_{-h_k}^{0} x(\tau)d\tau\right\}\\
&\quad=\operatorname{tr}\left(X^{-1}\mathcal{E}\left\{\sum_{k=1}^{m}\int_{-h_k}^{0} x(\tau)d\tau\left(\int_{-h_k}^{0} x(\tau)d\tau\right)^T\right\}\right)\\
&\quad=\operatorname{tr}\left(X^{-1}\mathcal{Z}_3\mathcal{Z}_3^T\right)<\operatorname{tr}(\Gamma_3). \qquad (30)
\end{aligned}$$

Hence, it follows from (26) that

$$J^* < \operatorname{tr}(\Gamma_0)+\operatorname{tr}(\Gamma_1)+\operatorname{tr}(\Gamma_2)+\operatorname{tr}(\Gamma_3).$$

Then, the minimization of $\operatorname{tr}(\Gamma_0)+\operatorname{tr}(\Gamma_1)+\operatorname{tr}(\Gamma_2)+\operatorname{tr}(\Gamma_3)$ implies the minimization of the guaranteed cost for the stochastic fuzzy system (6). The optimality of the solution of the optimization problem (26) follows from the convexity of the objective function and of the constraints.

This completes the proof. ■

In the preceding discussion, we presented sufficient conditions for delay-dependent guaranteed cost control of stochastic fuzzy systems with multiple time delays. When $k=1$, simpler results can be obtained in parallel to Theorems 1 and 2.

The closed-loop stochastic fuzzy system with single delay is described as follows:

$$\begin{aligned}
dx(t)=&\left(B_{b0}(\delta)+\Delta B_{b0}(\delta)\right)x(t)dt+\left(B_{b1}(\delta)+\Delta B_{b1}(\delta)\right)\\
&\times x(t-h)dt+\left(C_{b0}(\delta)+\Delta C_{b0}(\delta)\right)x(t)dw(t)\\
&+\left(C_{b1}(\delta)+\Delta C_{b1}(\delta)\right)x(t-h)dw(t) \qquad (31)
\end{aligned}$$

where

$$\begin{aligned}
B_{b0}(\delta)+\Delta B_{b0}(\delta)&=\sum_{i=1}^{r}\sum_{j=1}^{r}\delta_i\left(z(t)\right)\delta_j\left(z(t)\right)\\
&\quad\times\left(B_{i0}+\Delta B_{i0}(t)+D_iK_j\right)\\
B_{b1}(\delta)+\Delta B_{b1}(\delta)&=\sum_{i=1}^{r}\sum_{j=1}^{r}\delta_i\left(z(t)\right)\delta_j\left(z(t)\right)\\
&\quad\times\left(B_{i1}+\Delta B_{i1}(t)\right)\\
&=\sum_{i=1}^{r}\delta_i\left(z(t)\right)\left(B_{i1}+\Delta B_{i1}(t)\right)\\
C_{b0}(\delta)+\Delta C_{b0}(\delta)&=\sum_{i=1}^{r}\sum_{j=1}^{r}\delta_i\left(z(t)\right)\delta_j\left(z(t)\right)\\
&\quad\times\left(C_{i0}+\Delta C_{i0}(t)\right)\\
&=\sum_{i=1}^{r}\delta_i\left(z(t)\right)\left(C_{i0}+\Delta C_{i0}(t)\right)\\
C_{b1}(\delta)+\Delta C_{b1}(\delta)&=\sum_{i=1}^{r}\sum_{j=1}^{r}\delta_i\left(z(t)\right)\delta_j\left(z(t)\right)\\
&\quad\times\left(C_{i1}+\Delta C_{i1}(t)\right)\\
&=\sum_{i=1}^{r}\delta_i\left(z(t)\right)\left(C_{i1}+\Delta C_{i1}(t)\right).
\end{aligned}$$

B_{i0}, B_{i1}, C_{i0}, C_{i1}, and D_i are known constant matrices with compatible dimensions; K_i, $i=1,\ldots,r$, are control gain matrices, which are defined in (5); and the matrix functions $\Delta B_{i0}(t)$, $\Delta B_{i1}(t)$, $\Delta C_{i0}(t)$, and $\Delta C_{i1}(t)$ represent norm-bounded parameter uncertainties and satisfy

$$\begin{aligned}
&[\Delta B_{i0}(t)\quad \Delta B_{i1}(t)\quad \Delta C_{i0}(t)\quad \Delta C_{i1}(t)]\\
&\qquad=M_iF_i(t)\left[N_{1i0}\quad N_{1i1}\quad N_{2i0}\quad N_{2i1}\right] \qquad (32)
\end{aligned}$$

where M_i, N_{1i0}, N_{1i1}, N_{2i0}, and N_{2i1} are known constant matrices with compatible dimensions, and $F_i(t)$ is as defined in (2).

Corollary 1: Given $h>0$, the closed-loop stochastic fuzzy system (31) is asymptotically stable in the mean-square sense if there exist matrices $X>0$, $\hat{Q}>0$, $\hat{R}>0$, and $\hat{K}_i$ $(i=1,\ldots,r)$ with compatible dimensions and scalars $\varepsilon_{i1}>0$ and $\varepsilon_{i2}>0$, such that the following LMIs hold for $1\le i\le j\le r$:

$$\begin{bmatrix}
\Pi_{1,ij} & \Pi_{2,ij} & \Pi_{4,ij} & \Pi_{7,ij} & \Pi_{9,ij} & \Pi_{11,ij}\\
* & \Pi_{3,ij} & \Pi_{5,ij} & \Pi_{7,ij}^1 & \Pi_{9,ij}^1 & 0\\
* & * & \Pi_{6,ij} & 0 & 0 & 0\\
* & * & * & \Pi_{8,ij} & 0 & 0\\
* & * & * & * & \Pi_{10,ij} & 0\\
* & * & * & * & * & -\Pi_{12,ij}
\end{bmatrix}<0 \qquad (33)$$

where

$$\begin{aligned}
\Pi_{1,ij} &= B_{i0}X + XB_{i0}^T + B_{j0}X + XB_{j0}^T + D_j\hat{K}_i + D_i\hat{K}_j \\
&\quad + \hat{K}_i^T D_j^T + \hat{K}_j^T D_i^T + \varepsilon_{i1}M_iM_i^T + \varepsilon_{j1}M_jM_j^T \in R^{n\times n} \\
\Pi_{2,ij} &= B_{i1}X + B_{j1}X \in R^{n\times n} \\
\Pi_{3,ij} &= -4X + 2\hat{Q} \in R^{n\times n} \\
\Pi_{4,ij} &= 2hX \in R^{n\times n} \\
\Pi_{5,ij} &= -2hX \in R^{n\times n} \\
\Pi_{6,ij} &= -4hX + 2h\hat{R} \in R^{n\times n} \\
\Pi_{7,ij} &= \begin{bmatrix} XC_{i0}^T & XC_{j0}^T & XN_{2i0}^T & XN_{2j0}^T \end{bmatrix} \in R^{n\times(2n+2f)} \\
\Pi_{7,ij}^1 &= \begin{bmatrix} XC_{i1}^T & XC_{j1}^T & XN_{2i1}^T & XN_{2j1}^T \end{bmatrix} \in R^{n\times(2n+2f)} \\
\Pi_{8,ij} &= -\mathrm{diag}\left(X - \varepsilon_{i2}M_iM_i^T, X - \varepsilon_{j2}M_jM_j^T, \varepsilon_{i2}I, \varepsilon_{j2}I\right) \\
&\quad \in R^{(2n+2f)\times(2n+2f)} \\
\Pi_{9,ij} &= \begin{bmatrix} XN_{1i0}^T & XN_{1j0}^T \end{bmatrix} \in R^{n\times 2f} \\
\Pi_{9,ij}^1 &= \begin{bmatrix} XN_{1i1}^T & XN_{1j1}^T \end{bmatrix} \in R^{n\times 2f} \\
\Pi_{10,ij} &= -\mathrm{diag}(\varepsilon_{i1}I, \varepsilon_{j1}I) \in R^{2f\times 2f} \\
\Pi_{11,ij} &= \begin{bmatrix} 2hX & 2X & 2X & \hat{K}_i^T & \hat{K}_j^T \end{bmatrix} \in R^{n\times(3n+2q)} \\
\Pi_{12,ij} &= \mathrm{diag}(2h\hat{R}, 2\hat{Q}, 2\Xi^{-1}, \Psi^{-1}, \Psi^{-1}) \in R^{(3n+2q)\times(3n+2q)}.
\end{aligned}$$

Moreover, the control gain matrix can be chosen as $K_i = \hat{K}_i X^{-1}$, and the guaranteed cost bound is given by

$$\begin{aligned}
J^* &= \mathcal{E}\left\{x^T(0)X^{-1}x(0)\right\} + \mathcal{E}\left\{\int_{-h}^{0} x^T(\tau)\hat{Q}^{-1}x(\tau)d\tau\right\} \\
&\quad + \mathcal{E}\left\{\int_{-h}^{0}\int_{\beta}^{0} x^T(\tau)\hat{R}^{-1}x(\tau)d\tau d\beta\right\} \\
&\quad + \mathcal{E}\left\{\left(\int_{-h}^{0} x(\tau)d\tau\right)^T X^{-1}\int_{-h}^{0} x(\tau)d\tau\right\}. \qquad (34)
\end{aligned}$$

Corollary 2: Consider the stochastic fuzzy system (31) with cost function (7). If the following optimization problem:

$$\min\left\{\mathrm{tr}(\Gamma_0) + \mathrm{tr}(\Gamma_1) + \mathrm{tr}(\Gamma_2) + \mathrm{tr}(\Gamma_3)\right\}$$

$$\text{s.t.}\begin{cases}
\text{(i)} & \text{inequality (33);} \\
\text{(ii)} & \begin{bmatrix} -\Gamma_0 & \mathcal{Z}_0^T \\ \mathcal{Z}_0 & -X \end{bmatrix} < 0 \\
\text{(iii)} & \begin{bmatrix} -\Gamma_1 & \mathcal{Z}_1^T \\ \mathcal{Z}_1 & -\hat{R} \end{bmatrix} < 0 \\
\text{(iv)} & \begin{bmatrix} -\Gamma_2 & \mathcal{Z}_2^T \\ \mathcal{Z}_2 & -\hat{Q} \end{bmatrix} < 0 \\
\text{(v)} & \begin{bmatrix} -\Gamma_3 & \mathcal{Z}_3^T \\ \mathcal{Z}_3 & -X \end{bmatrix} < 0
\end{cases} \qquad (35)$$

has a solution set $\Theta = (\varepsilon_{i1}, \varepsilon_{i2}, X, \hat{R}, \hat{Q}, \hat{K}_i, \Gamma_0, \Gamma_1, \Gamma_2, \Gamma_3, 1 \le i \le j \le r)$, then controller (5) is an optimal guaranteed cost controller, which ensures the minimization of the guaranteed cost bound (34) for system (31), where

$$\begin{aligned}
\mathcal{Z}_0\mathcal{Z}_0^T &= \mathcal{E}\left\{x(0)x^T(0)\right\} \\
\mathcal{Z}_1\mathcal{Z}_1^T &= \mathcal{E}\left\{\int_{-h}^{0}\int_{\beta}^{0} x(\tau)x^T(\tau)d\tau d\beta\right\} \\
\mathcal{Z}_2\mathcal{Z}_2^T &= \mathcal{E}\left\{\int_{-h}^{0} x(\tau)x^T(\tau)d\tau\right\} \\
\mathcal{Z}_3\mathcal{Z}_3^T &= \mathcal{E}\left\{\int_{-h}^{0} x(\tau)d\tau\left(\int_{-h}^{0} x(\tau)d\tau\right)^T\right\}.
\end{aligned}$$

Remark 2: We have presented delay-dependent sufficient conditions for guaranteed cost control in terms of the convex LMI format. Next, we make a comparison with the existing delay-dependent results in [12]. Guan and Chen [12] have pointed out that some existing approaches cannot provide sufficient conditions based on the convex LMI format; furthermore, the global minimum of the aforementioned minimization problem cannot be found using a convex optimization algorithm, and the suboptimal solutions have to be chosen. Therefore, their approach may lead to a heavy computational burden. However, the approach in this paper can lead to convex LMI conditions such that the global minimum solution can be directly solved by the LMI toolbox in Matlab. Therefore, our approach not only reduced the computational cost of solution process but also enhanced the control performance of the closed-loop system.

IV. Illustrative Examples

In this section, a system with a single time delay and a system with two time delays are used to illustrate the effectiveness of the present approach in Examples 1 and 2, respectively.

Example 1: Consider the following stochastic nonlinear delayed system:

$$\begin{aligned}
dx_1(t) &= \big(-0.1125x_1(t) - 0.0125x_1(t-0.3) - 0.02x_2(t) \\
&\quad - 0.67x_2^3(t) - 0.005x_2(t-0.3) + u(t)\big)\,dt \\
&\quad + \left(0.5x_1(t) - 0.4x_2(t) + 0.4x_2^3(t)\right)d\omega(t) \\
dx_2(t) &= x_1(t)dt + \left(0.15x_1(t) + 0.9x_2(t) + 0.4x_2^3(t)\right)d\omega(t).
\end{aligned} \qquad (36)$$

Similar to [19], assume that $x_1(t)$ and $x_2(t)$ are measurable and $x_1(t) \in [-1.5, 1.5]$ and $x_2(t) \in [-1.5, 1.5]$. The nonlinear terms of system can be represented as

$$-0.67x_2^3(t) = \mathcal{R}_{11}(x_2(t))\cdot 0\cdot x_2(t) - \mathcal{R}_{12}(x_2(t))\cdot 1.5075x_2(t)$$

$$0.4x_2^3(t) = \mathcal{R}_{11}(x_2(t))\cdot 0\cdot x_2(t) - \mathcal{R}_{12}(x_2(t))\cdot(-0.9)x_2(t).$$

TABLE I
DATA OF 100 EXPERIMENTS FOR THE GUARANTEED COST VALUE

times	state(x_1)	J	times	state(x_1)	J	times	state(x_1)	J
1	0.2311	1.1269	36	0.8462	15.0957	71	0.3093	2.0204
2	0.6068	7.7764	37	0.5252	5.8272	72	0.8385	14.8212
3	0.486	4.9866	38	0.2026	0.8675	73	0.5681	6.8208
4	0.8913	16.7895	39	0.6721	9.5452	74	0.3704	2.897
5	0.7621	12.2611	40	0.8381	14.8079	75	0.7027	10.4303
6	0.4565	4.4024	41	0.0196	8.15E-03	76	0.5466	6.3121
7	0.0185	0.0072	42	0.6813	9.8106	77	0.4449	4.1794
8	0.8214	14.2521	43	0.3795	3.0418	78	0.6946	10.1967
9	0.4447	4.1761	44	0.8318	14.6148	79	0.6213	8.168
10	0.6154	7.9969	45	0.5028	5.3412	80	0.7948	13.3405
11	0.7919	13.2439	46	0.7095	10.6305	81	0.9568	19.3499
12	0.9218	17.9634	47	0.4289	3.8887	82	0.5226	5.7696
13	0.7382	11.5129	48	0.3046	1.9637	83	0.8801	16.3726
14	0.1763	0.6567	49	0.1897	0.759	84	0.173	0.6312
15	0.4057	3.4768	50	0.1934	0.7899	85	0.9797	20.2531
16	0.9355	18.4947	51	0.6822	9.8379	86	0.2714	1.5566
17	0.9169	17.7728	52	0.3028	1.9355	87	0.2523	1.3441
18	0.4103	3.5556	53	0.5417	6.1991	88	0.8757	16.2097
19	0.8936	16.878	54	0.1509	0.4805	89	0.7373	11.4848
20	0.0579	0.0707	55	0.6979	10.2873	90	0.1365	0.3936
21	0.3529	2.6291	56	0.3784	3.0241	91	0.0118	0.0029
22	0.8132	13.9802	57	0.86	15.5916	92	0.8939	16.8873
23	0.0099	0.002	58	0.8537	15.362	93	0.1991	0.8378
24	0.1389	0.4077	59	0.5936	7.4293	94	0.2987	1.8826
25	0.2028	0.8685	60	0.4966	5.205	95	0.6614	9.2412
26	0.1987	0.8343	61	0.8998	17.1122	96	0.2844	1.709
27	0.6038	7.6984	62	0.8216	14.2598	97	0.4692	4.6531
28	0.2722	1.5638	63	0.6449	8.7872	98	0.0648	0.0886
29	0.1988	0.8351	64	0.818	14.1422	99	0.9883	20.63
30	0.0153	0.0049	65	0.6602	9.2069	100	0.5828	7.1674
31	0.7468	11.7652	66	0.342	2.4699			
32	0.4451	4.1835	67	0.2897	1.7737			
33	0.9318	18.3504	68	0.3412	2.4587			
34	0.466	4.5886	69	0.5341	6.0271			
35	0.4186	3.6973	70	0.7271	11.1701			

Solving these equations, we obtain

$$\mathcal{R}_{11}\left(x_2(t)\right) = 1 - x_2^2(t)/2.25$$

$$\mathcal{R}_{12}\left(x_2(t)\right) = 1 - \mathcal{R}_{11}\left(x_2(t)\right) = x_2^2(t)/2.25$$

where $\mathcal{R}_{11}(x_2(t))$ and $\mathcal{R}_{12}(x_2(t))$ can be interpreted as membership functions of fuzzy sets. Using these fuzzy sets, the stochastic nonlinear system with time delay can be expressed by the following stochastic fuzzy model:

Rule 1 : IF $x_2(t)$ is $\mathcal{R}_{11}$

THEN

$$\begin{aligned} dx(t) &= \left[\left(B_{10}+\Delta B_{10}(t)\right)x(t)+B_{11}x(t-h)\right]dt \\ &\quad + D_1 u(t)dt+\left(C_{10}+\Delta C_{10}(t)\right)x(t)dw(t) \end{aligned}$$

Rule 2 : IF $x_2(t)$ is $\mathcal{R}_{12}$

THEN

$$\begin{aligned} dx(t) &= \left[\left(B_{20}+\Delta B_{20}(t)\right)x(t)+B_{21}x(t-h)\right]dt \\ &\quad + D_2 u(t)dt+\left(C_{20}+\Delta C_{20}(t)\right)x(t)dw(t) \end{aligned}$$

where $x(t) = [x_1(t) \quad x_2(t)]^T$.

System parameters B_{10}, B_{20}, B_{11}, B_{21}, C_{10}, C_{20}, D_1, and D_2 can be solved by the following equations:

$$\begin{aligned} &\begin{bmatrix} -0.1125x_1(t) - 0.02x_2(t) - 0.67x_2^3 \\ x_1 \end{bmatrix} \\ &= \mathcal{R}_{11}(x_2(t))B_{10}x(t) + \mathcal{R}_{12}(x_2(t))B_{20}x(t) \\ &= \left(1 - \frac{x_2^2(t)}{2.25}\right)B_{10}x(t) + \frac{x_2^2(t)}{2.25}B_{20}x(t) \end{aligned}$$

$$\begin{aligned} &\begin{bmatrix} 0.5x_1(t) - 0.4x_2(t) + 0.4x_2^3(t) \\ 0.15x_1(t) + 0.9x_2(t) + 0.4x_2^3(t) \end{bmatrix} \\ &= \mathcal{R}_{11}\left(x_2(t)\right)C_{10}x(t) + \mathcal{R}_{12}\left(x_2(t)\right)C_{20}x(t) \\ &= \left(1 - \frac{x_2^2(t)}{2.25}\right)C_{10}x(t) + \frac{x_2^2(t)}{2.25}C_{20}x(t) \end{aligned}$$

$$\begin{aligned} &\begin{bmatrix} -0.0125x_1(t-0.3) - 0.005x_2(t-0.3) \\ 0 \end{bmatrix} \\ &= \mathcal{R}_{11}\left(x_2(t)\right)B_{11}x(t-0.3) + \mathcal{R}_{12}\left(x_2(t)\right)B_{21}x(t-0.3) \end{aligned}$$

$$\begin{bmatrix} u(t) \\ 0 \end{bmatrix} = \mathcal{R}_{11}\left(x_2(t)\right)D_1 u(t) + \mathcal{R}_{12}\left(x_2(t)\right)D_2 u(t).$$

TABLE II
SOME STATISTICS OF THE GUARANTEED COST PERFORMANCE OF REGULATION

min(J)	max(J)	ave(J)	std(J)	median(J)
0.002	20.63	7.4421	6.1427	5.9271

Then, we have

$$B_{10} = \begin{bmatrix} -0.1125 & -0.02 \\ 1 & 0 \end{bmatrix} \quad C_{10} = \begin{bmatrix} 0.5 & -0.4 \\ 0.15 & 0.9 \end{bmatrix}$$

$$B_{20} = \begin{bmatrix} -0.1125 & -1.5275 \\ 1 & 0 \end{bmatrix} \quad C_{20} = \begin{bmatrix} 0.5 & 0.5 \\ 0.15 & 1.8 \end{bmatrix}$$

$$B_{11} = B_{21} = \begin{bmatrix} -0.0125 & -0.005 \\ 0 & 0 \end{bmatrix} \quad D_1 = D_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$[\,\Delta B_{10}(t) \quad \Delta C_{10}(t)\,] = M_1 F(t)\,[\,N_{110} \quad N_{210}\,]$$

$$[\,\Delta B_{20}(t) \quad \Delta C_{20}(t)\,] = M_2 F(t)\,[\,N_{120} \quad N_{220}\,]$$

$$M_1 = M_2 = \begin{bmatrix} -0.1125 \\ 0 \end{bmatrix} \quad N_{110} = N_{120} = [\,1 \quad 0\,]$$

$$N_{210} = N_{221} = [\,0 \quad 1\,].$$

The time delay is $h = 0.3$. Assume that the initial function $x_1(t)$ is a random constant value in [0,1] and that $x_2(t) = 0$ for $t \in [-0.3,\ 0]$ and uncertain function $F(t) = \sin(t)$.

Remark 3: In the preceding T-S model, uncertain parameters ΔB_{10}, ΔB_{20}, ΔC_{10}, and ΔC_{20} are introduced, because we consider the robust control performance of the system. The form of uncertain function $F(t)$ does not affect the stability result of the robust control systems as long as it satisfies the condition $F^T(t)F(t) \le I$. ■

Given $\Xi = \mathrm{diag}(1, 1)$ and $\Psi = 1$, applying Corollary 2, we performed 100 experiments and presented the corresponding data in Table I. Table II provides the statistic analysis performances of guaranteed cost function value J.

The computation formulas are given as follows:

$$\text{maximum value } \max(J) = \max_{1 \le i \le 100} (J_i)$$

$$\text{minimum value } \min(J) = \min_{1 \le i \le 100} (J_i)$$

$$\text{average value } \mathrm{ave}(J) = \left(\sum_{i=1}^{100} J_i \right) \Big/ 100$$

$$\text{standard deviation } \mathrm{std}(J) = \sqrt{\left(\sum_{i=1}^{100} (J_i - \mathrm{ave}(J))^2 \right) \Big/ 100}$$

and median value $\mathrm{med}(J)$ can be solved in two steps.

1) Arrange those values of J_i, $i = 1, \ldots, 100$, from minimum to maximum: $J^1 \le J^2 \le \cdots \le J^{50} \le J^{51} \le J^{100}$.
2) Then, $\mathrm{med}(J) = (J^{50} + J^{51})/2$.

When the initial function $x_1(t) = 0.5$ and $x_2(t) = 0$ for $t \in [-0.3,\ 0]$ and $\Xi = \mathrm{diag}(1, 1)$ and $\Psi = 1$, based on Corollary 2 again, we can get $\varepsilon_{11} = 0.1092$, $\varepsilon_{12} = 0.2534$, $\varepsilon_{21} = 1.2320$, $\varepsilon_{22} = 1.3813$, $X = \begin{bmatrix} 0.2928 & -0.0622 \\ -0.0622 & 0.0226 \end{bmatrix}$, $\hat{R} = \begin{bmatrix} 0.2473 & -0.0474 \\ -0.0474 & 0.0168 \end{bmatrix}$, $\hat{Q} = \begin{bmatrix} 0.4172 & -0.0874 \\ -0.0874 & 0.0314 \end{bmatrix}$, $\hat{K}_1 = [-0.9975 \quad -0.0010]$, and $\hat{K}_2 = [-0.9998 \quad -0.0001]$, with the resulting control gain matrices $K_1 = [-8.2178 \; -22.6455]$ and $K_2 = [-8.2159 \quad -22.5998]$.

The simulation results on guaranteed cost control based on state feedback are shown in Fig. 1. With control law $u(t) = \mathcal{R}_{11}(x_2(t))K_1 x(t) + \mathcal{R}_{12}(x_2(t))K_2 x(t)$, the closed-loop system is asymptotically stable in the mean-square sense, and the guaranteed cost bound is $J^* = 5.1292$. Fig. 2 shows the curve of the control signal.

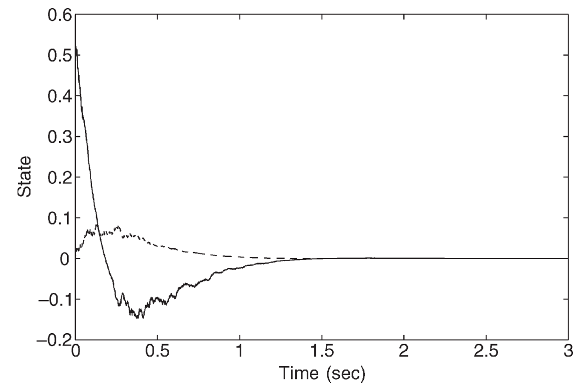

Fig. 1. Response of state via state feedback (solid line: x_1, dashed line: x_2).

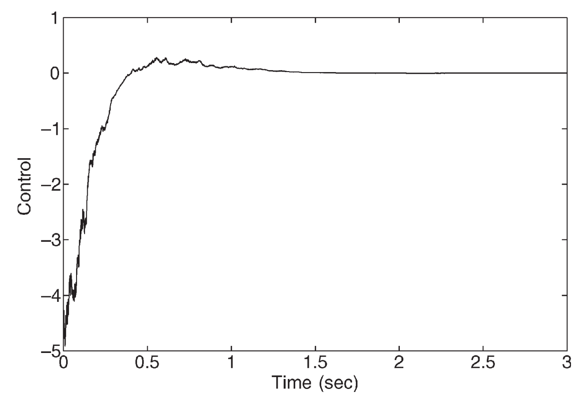

Fig. 2. Curve of control input via state feedback.

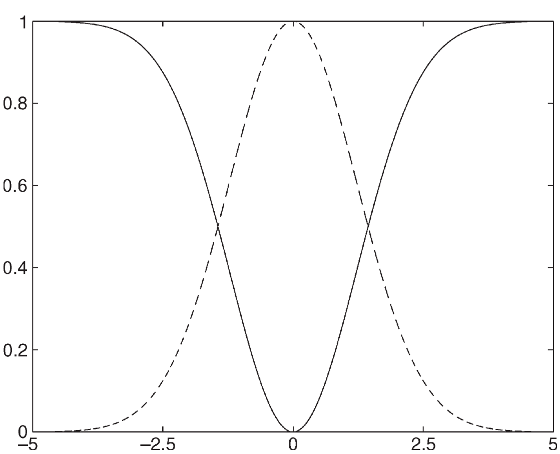

Fig. 3. Membership functions (dashed line: rule 1, solid line: rule 2).

TABLE III
DATA OF 100 EXPERIMENTS FOR THE GUARANTEED COST VALUE

times	State(x_1)	State(x_2)	J		times	State(x_1)	State(x_2)	J
1	0.8658	0.4519	235.6765		51	0.5948	0.0983	70.7521
2	1.0596	1.2811	737.419		52	1.3864	1.3002	966.4405
3	0.4181	0.7596	191.1987		53	1.966	1.1053	1273.067
4	1.5667	1.3617	1147.266		54	0.7518	0.0198	94.7339
5	1.7487	0.03	508.432		55	1.5877	1.8399	1576.808
6	1.5359	1.9417	1631.082		56	1.6894	0.7355	805.0326
7	1.9802	1.5777	1693.996		57	1.2416	1.4626	980.8259
8	0.8773	0.9966	472.337		58	0.3878	1.8096	719.93
9	0.6401	1.9202	941.1683		59	1.1384	1.2636	773.016
10	1.4533	0.8239	700.4928		60	0.4688	1.0976	344.4478
11	1.4891	0.5359	570.168		61	1.8632	0.6704	890.66
12	0.8798	1.8668	1052.442		62	1.2546	1.3982	942.5434
13	1.3667	0.4251	451.305		63	0.7944	0.8273	352.903
14	1.2144	1.2598	821.372		64	1.3104	1.6752	1203.796
15	0.741	1.1503	486.833		65	1.1893	1.1315	722.035
16	0.0544	0.6254	73.234		66	1.4331	1.0226	809.0723
17	0.0257	0.7679	103.276		67	1.5528	0.9787	862.8219
18	1.3662	0.1857	359.938		68	0.3718	1.4013	460.984
19	0.0707	1.2248	270.901		69	1.4071	0.9699	758.4267
20	1.2171	0.0315	248.523		70	0.2292	1.3297	370.031
21	0.0327	0.3801	27.079		71	0.7307	0.2801	141.354
22	1.1738	0.1152	253.273		72	1.3479	1.9989	1523.01
23	1.4353	1.3853	1064.923		73	1.9233	0.1177	647.356
24	0.1682	0.9087	174.953		74	0.7206	1.097	449.635
25	0.3072	1.3513	408.484		75	0.5235	1.1947	414.094
26	1.3984	1.455	1090.517		76	0.0986	1.1421	244.3782
27	0.2421	0.9015	191.284		77	1.4017	1.9246	1496.301
28	1.4318	1.7857	1398.068		78	1.501	1.48	1192.496
29	0.5462	0.5095	149.198		79	0.8637	1.2685	616.495
30	1.7312	0.4647	684.595		80	1.6061	0.1678	477.596
31	1.6097	1.8168	1577.857		81	1.8909	1.8319	1853.929
32	0.0995	0.1568	8.9454		82	1.204	0.5071	402.6291
33	1.6877	0.3478	601.345		83	1.7469	1.0268	1037.628
34	0.3416	1.9886	824.404		84	1.4653	0.8445	720.088
35	0.8796	0.6801	325.724		85	1.9227	0.1441	658.77
36	0.6284	0.7302	248.445		86	1.1068	0.584	387.149
37	0.7865	1.1831	527.1755		87	1.3604	0.1069	331.837
38	0.9172	1.7397	977.1687		88	0.7133	0.9966	396.28
39	1.8685	0.5289	812.154		89	0.8689	1.1249	537.551
40	0.3206	1.7457	645.1055		90	1.2332	0.2267	311.357
41	0.4758	1.2917	444.194		91	1.7965	1.5091	1460.503
42	1.9338	1.3299	1429.081		92	1.5822	1.6299	1385.145
43	1.7408	0.0199	499.615		93	1.34	0.4018	427.489
44	0.274	1.6375	556.103		94	0.5462	1.2525	454.3748
45	0.6922	0.3321	143.01		95	1.0737	0.119	214.7259
46	0.3112	0.3822	64.716		96	0.1779	0.5426	74.7015
47	0.8449	1.712	908.999		97	1.818	1.1925	1219.259
48	0.9805	1.6319	933.7022		98	1.1943	0.3229	326.509
49	0.9014	0.8244	398.1939		99	1.6242	1.2202	1082.588
50	1.8032	0.0112	532.406		100	1.403	0.1844	377.392

Example 2: Consider a stochastic system with two time delays that is described by two fuzzy rules as follows:

Rule 1 : IF $x_1(t)$ is $\mathcal{R}_{11}$

THEN

$$dx(t)=\sum_{k=0}^{2}(B_{1k}+\Delta B_{1k}(t))\,x(t-h_k)dt+D_1u(t)dt+\sum_{k=0}^{2}(C_{1k}+\Delta C_{1k}(t))\,x(t-h_k)dw(t)$$

Rule 2 : IF $x_1(t)$ is $\mathcal{R}_{12}$

THEN

$$dx(t)=\sum_{k=0}^{2}(B_{2k}+\Delta B_{2k}(t))\,x(t-h_k)dt+D_2u(t)dt+\sum_{k=0}^{2}(C_{2k}+\Delta C_{2k}(t))\,x(t-h_k)dw(t)$$

where $x(t)=[x_1(t)\quad x_2(t)]^T$. The fuzzy membership functions $\mathcal{R}_{11}$ and $\mathcal{R}_{12}$ are defined as in Fig. 3. The system parameters are given as

$$B_{10}=\begin{bmatrix}-7.3 & 0.1\\ 0.3 & -7.4\end{bmatrix}\quad B_{11}=\begin{bmatrix}-0.1 & -0.01\\ 0.01 & -0.1\end{bmatrix}$$

$$B_{12}=\begin{bmatrix}-0.1 & -0.01\\ 0.01 & -0.1\end{bmatrix}\quad C_{10}=\begin{bmatrix}0.1 & -0.1\\ 0.2 & 0.5\end{bmatrix}$$

$$C_{11}=\begin{bmatrix}-0.2 & -0.3\\ 0.1 & -0.5\end{bmatrix}\quad C_{12}=\begin{bmatrix}-0.2 & -0.3\\ 0.1 & -0.5\end{bmatrix}$$

$$B_{20}=\begin{bmatrix}-7.3 & -0.4\\ 0.3 & -8.9\end{bmatrix}\quad B_{21}=\begin{bmatrix}-0.09 & 0.02\\ 0.01 & -0.1\end{bmatrix}$$

$$B_{22}=\begin{bmatrix}-0.09 & 0.02\\ 0.01 & -0.1\end{bmatrix}\quad C_{20}=\begin{bmatrix}0.1 & 0.1\\ 0.3 & -0.6\end{bmatrix}$$

$$C_{21}=\begin{bmatrix}0.3 & -0.2\\ 0.2 & 0.6\end{bmatrix}\quad C_{22}=\begin{bmatrix}0.3 & -0.2\\ 0.2 & 0.6\end{bmatrix}$$

$$D_1=D_2=\begin{bmatrix}1\\ 0.5\end{bmatrix}$$

TABLE IV
STATISTICS ANALYSIS OF THE GUARANTEED COST PERFORMANCE OF REGULATION

min(J)	max(J)	ave(J)	std(J)	median(J)
8.9454	1853.929	664.3873	442.2.31	546.827

and uncertain parameters are described by

$$[\Delta B_{10}(t)\ \ \Delta B_{11}(t)\ \ \Delta B_{12}(t)\ \ \Delta C_{10}(t)\ \ \Delta C_{11}(t)\ \ \Delta C_{12}(t)]$$
$$= M_1F_1(t)[N_{110}\ \ N_{111}\ \ N_{112}\ \ N_{210}\ \ N_{211}\ \ N_{212}]$$
$$[\Delta B_{20}(t)\ \ \Delta B_{21}(t)\ \ \Delta B_{22}(t)\ \ \Delta C_{20}(t)\ \ \Delta C_{21}(t)\ \ \Delta C_{22}(t)]$$
$$= M_2F_2(t)[N_{120}\ \ N_{121}\ \ N_{122}\ \ N_{220}\ \ N_{221}\ \ N_{222}]$$

with

$$M_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.5 \end{bmatrix} \quad N_{110} = N_{111} = N_{112} = \begin{bmatrix} 0.5 & 1 \\ -0.2 & 0.6 \end{bmatrix}$$
$$N_{210} = N_{211} = \begin{bmatrix} 1 & 0.5 \\ 0.2 & -0.6 \end{bmatrix} \quad M_2 = \begin{bmatrix} 0.2 & 0.3 \\ -0.3 & 0.4 \end{bmatrix}$$
$$N_{120} = N_{121} = N_{122} = \begin{bmatrix} -0.5 & 0.9 \\ 0.2 & -0.3 \end{bmatrix}$$
$$N_{220} = N_{221} = \begin{bmatrix} 0.9 & 0.5 \\ -0.2 & 0.3 \end{bmatrix} \quad N_{212} = N_{222} = [0]_{2\times 2}.$$

The time delays are $h_1 = 0.25$ and $h_2 = 0.5$. Assume that the initial function $\zeta(t) = [x_1(t)\ \ \ x_2(t)]^T$ is a random constant value in [0, 2] for $t \in [-0.5,\ \ 0]$ and uncertain functions $F_1(t) = F_2(t) = \sin(t)$. Applying Theorem 2, we performed 100 experiments and recorded the corresponding data in Table III. We can also calculate the statistics of guaranteed cost function value J as in Example 1, as shown in Table IV.

When $\zeta(t) = [x_1(t)\ x_2(t)]^T = [0.5\ \ 1]^T$ for $t \in [-0.5, 0]$, $\Xi = \mathrm{diag}(1, 1)$, and $\Psi = 1$, based on Theorem 2, a feasible solution is given as follows: $\varepsilon_{11} = 0.0516$, $\varepsilon_{12} = 1.0015$, $\varepsilon_{21} = 0.0.0629$, $\varepsilon_{22} = 0.6285$, $X = \begin{bmatrix} 0.0615 & -0.0166 \\ -0.0166 & 0.0658 \end{bmatrix}$, $\hat{R} = \begin{bmatrix} 0.0253 & -0.0142 \\ -0.0142 & 0.0220 \end{bmatrix}$, $\hat{Q} = \begin{bmatrix} 0.0198 & -0.0126 \\ -0.0126 & 0.0198 \end{bmatrix}$, $\hat{K}_1 = [-0.9998\ \ -0.5000]$, and $\hat{K}_2 = [-0.9925\ \ -0.4973]$, with the resulting control gain matrices $K_1 = [-19.6567\ \ -12.5763]$ and $K_2 = [-19.5165\ \ -12.4999]$.

The simulation results of the guaranteed cost control based on state feedback are shown in Fig. 4. With control law $u(t) = \mathcal{R}_{11}(x_2(t))K_1x(t) + \mathcal{R}_{12}(x_2(t))K_2x(t)$, the closed-loop system is asymptotically stable in the mean-square sense, and the guaranteed cost bound is $J^* = 311.8885$. Fig. 5 shows the control curve. It is easy to see that all the time responses of states are satisfactory.

V. CONCLUSION

In this paper, a class of uncertain stochastic fuzzy systems with multiple time delays is studied. A delay-dependent guaranteed cost control approach was developed such that the designed state feedback controller can guarantee that the closed-loop system is asymptotically stable in the mean-square sense and the value of cost function is not larger than a bound. The present approach does not require system transformation or relaxation matrices. All results were presented in the solvable form of LMIs. Simulation examples were given to illustrate the design procedures and the effectiveness of the approach.

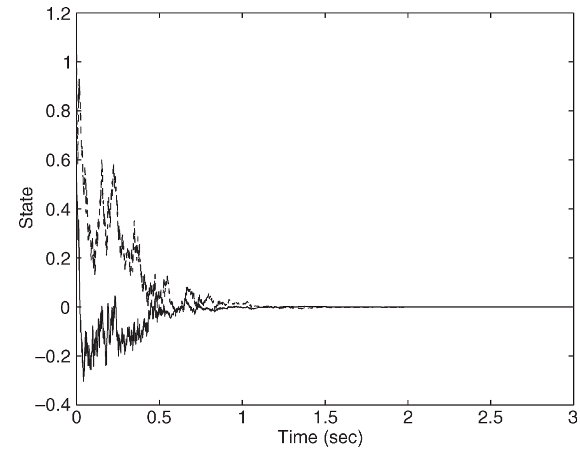

Fig. 4. Response of state via state feedback (solid line: x_1, dashed line: x_2).

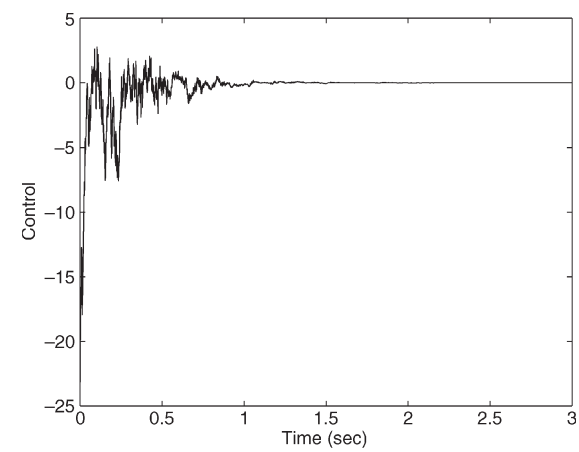

Fig. 5. Curve of control input via state feedback.

APPENDIX

Proof of Lemma 2: Since W is a positive definite symmetric constant matrix, for any nonzero $c = [c_1, c_2, \ldots, c_{m+1}]^T \triangleq [c_1, q^T]^T$, we have $(c_1\bar{v}(s) + W^{-1}q)^T \times W(c_1\bar{v}(s) + W^{-1}q) \geq 0$, i.e., $c_1\bar{v}^T(s)W(c_1\bar{v}(s)) + c_1q^T\bar{v}(s) + c_1\bar{v}^T(s)q + q^TW^{-1}q \geq 0$, which is equivalent to

$$c^T \begin{bmatrix} \bar{v}^T(s)W\bar{v}(s) & \bar{v}^T(s) \\ \bar{v}(s) & W^{-1} \end{bmatrix} c \geq 0. \tag{37}$$

Since c is an arbitrary nonzero vector, from inequality (37), it follows that

$$\begin{bmatrix} \bar{v}^T(s)W\bar{v}(s) & \bar{v}^T(s) \\ \bar{v}(s) & W^{-1} \end{bmatrix} \geq 0. \tag{38}$$

Integrating (38) from $\beta-\kappa$ to β yields

$$\begin{bmatrix} \int_{\beta-\kappa}^{\beta} \bar{v}^T(s)W\bar{v}(s)ds & \left(\int_{\beta-\kappa}^{\beta} \bar{v}(s)ds\right)^T \\ \int_{\beta-\kappa}^{\beta} \bar{v}(s)ds & \kappa W^{-1} \end{bmatrix} \geq 0.$$

Using the Schur complement, we have

$$\int_{\beta-\kappa}^{\beta} \bar{v}^T(s)W\bar{v}(s)ds - \frac{1}{\kappa}\left(\int_{\beta-\kappa}^{\beta} \bar{v}(s)ds\right)^T W \int_{\beta-\kappa}^{\beta} \bar{v}(s)ds \geq 0$$

or

$$\kappa \int_{\beta-\kappa}^{\beta} \bar{v}^T(s)W\bar{v}(s)ds \geq \left(\int_{\beta-\kappa}^{\beta} \bar{v}(s)ds\right)^T W \int_{\beta-\kappa}^{\beta} \bar{v}(s)ds.$$

This completes the proof. ■

REFERENCES

[1] S. Boyd, L. El Ghaoul, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Philadelphia, PA: SIAM, 1994.

[2] Y. Y. Cao and Y. X. Sun, "Robust stabilization of uncertain systems with time-varying multi-state-delay," *IEEE Trans. Autom. Control*, vol. 43, no. 10, pp. 1484–1488, Oct. 1998.

[3] S. S. L. Chang and T. K. C. Peng, "Adaptive guaranteed cost control of systems with uncertain parameters," *IEEE Trans. Autom. Control*, vol. AC-17, no. 4, pp. 474–483, Aug. 1972.

[4] W. H. Chen, Z. H. Guan, and X. M. Lu, "Delay-dependent exponential stability of uncertain stochastic systems with multiple delays: An LMI approach," *Syst. Control Lett.*, vol. 54, no. 6, pp. 547–555, Jun. 2005.

[5] B. S. Chen, B. K. Lee, and L. B. Guo, "Optimal tracking design for stochastic fuzzy systems," *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 6, pp. 796–813, Dec. 2003.

[6] B. Chen and X. P. Liu, "Fuzzy guaranteed cost control for nonlinear systems with time-varying delay," *IEEE Trans. Fuzzy Syst.*, vol. 13, no. 2, pp. 238–249, Apr. 2005.

[7] B. S. Chen, C. S. Tseng, and H. J. Uang, "Fuzzy differential games for nonlinear stochastic systems: Suboptimal approach," *IEEE Trans. Fuzzy Syst.*, vol. 10, no. 2, pp. 222–233, Apr. 2002.

[8] P. Florchinger, "Lyapunov-like techniques for stochastic stability," *SIAM J. Control Optim.*, vol. 33, no. 4, pp. 1151–1169, Jul. 1995.

[9] E. Fridman and U. Shaked, "A descriptor system approach to H_∞ control of linear time-delay systems," *IEEE Trans. Autom. Control*, vol. 47, no. 2, pp. 253–270, Feb. 2002.

[10] K. Q. Gu, "An integral inequality in the stability problem of time-delay systems," in *Proc. 39th IEEE Conf. Decision Control*, Sydney, Australia, Dec. 2000, pp. 2805–2810.

[11] K. Q. Gu and S. I. Niculescu, "Additional dynamics in transformed time-delay systems," *IEEE Trans. Autom. Control*, vol. 45, no. 3, pp. 572–575, Mar. 2000.

[12] X. P. Guan and C. L. Chen, "Delay-dependent guaranteed cost control for T-S fuzzy systems with time delays," *IEEE Trans. Fuzzy Syst.*, vol. 12, no. 2, pp. 236–249, Apr. 2004.

[13] D. Hinrichsen and A. J. Pritchard, "Stochastic H_∞," *SIAM J. Control Optim.*, vol. 36, no. 5, pp. 1504–1538, Sep. 1998.

[14] C. Lin, Q. G. Wang, and T. H. Lee, "A less conservative robust stability test for linear uncertain time-delay systems," *IEEE Trans. Autom. Control*, vol. 51, no. 1, pp. 87–91, Jan. 2005.

[15] C. Lin, Q. G. Wang, and T. H. Lee, "H_∞ output tracking control for nonlinear systems via T-S fuzzy model approach," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 36, no. 2, pp. 450–457, Apr. 2006.

[16] X. Mao, *Stochastic Differential Equations and Applications*. Chichester, U.K.: Horwood, 1997.

[17] V. Suplin, E. Fridman, and U. Shaked, "H_∞ control of linear uncertain time-delay systems—A projection approach," *IEEE Trans. Autom. Control*, vol. 51, no. 4, pp. 680–685, Apr. 2006.

[18] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-15, no. 1, pp. 116–132, Jan. 1985.

[19] K. Tanaka, T. Ikeda, and H. O. Wang, "Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: Quadratic stabilizability, H_∞ control theory, and linear matrix inequalities," *IEEE Trans. Fuzzy Syst.*, vol. 4, no. 1, pp. 1–13, Feb. 1996.

[20] V. A. Ugrinovskii, "Output feedback guaranteed cost control for stochastic uncertain systems with multiplicative noise," in *Proc. 39th IEEE Conf. Decision Control*, Sydney, Australia, Dec. 2000, pp. 240–245.

[21] Z. D. Wang, D. W. C. Ho, and X. H. Liu, "A note on the robust stability of uncertain stochastic fuzzy systems with time-delays," *IEEE Trans. Syst., Man, Cybern. A, Syst., Humans*, vol. 34, no. 4, pp. 570–576, Jul. 2004.

[22] H. O. Wang, K. Tanaka, and M. F. Griffin, "Parallel distributed compensation of nonlinear systems by Takagi-Sugeno fuzzy model," in *Proc. IEEE Int. Conf. Fuzzy Syst.*, Yokohama, Japan, Mar. 1995, pp. 531–538.

[23] Y. Wang, L. Xie, and C. E. de Souza, "Robust control of a class of uncertain nonlinear systems," *Syst. Control Lett.*, vol. 19, no. 2, pp. 139–149, Aug. 1992.

[24] Y. C. Wang, H. G. Zhang, and Y. Z. Wang, "Fuzzy adaptive control of stochastic nonlinear systems with unknown virtual control gain function," *ACTA Autom. Sin.*, vol. 32, no. 2, pp. 170–178, Mar. 2006.

[25] W. M. Wonham, "Random differential equations in control theory," in *Probabilistic Methods in Applied Mathematics*, vol. 2. New York: Academic, 1970, pp. 131–217.

[26] S. Y. Xu and J. Lam, "Improved delay-dependent stability criteria for time-delay systems," *IEEE Trans. Autom. Control*, vol. 50, no. 3, pp. 384–387, Mar. 2005.

[27] E. E. Yaz, "Guaranteed-cost stabilization of non-linear stochastic systems," *Optim. Control Appl. Methods*, vol. 19, no. 1, pp. 41–54, 1998.

[28] L. Yu and J. Chu, "An LMI approach to guaranteed cost control of linear uncertain time-delay systems," *Automatica*, vol. 35, no. 6, pp. 1155–1159, Jun. 1999.

[29] D. Yue and Q. L. Han, "Delay-dependent exponential stability of stochastic systems with time-varying delay, nonlinearity, and Markovian switching," *IEEE Trans. Autom. Control*, vol. 50, no. 2, pp. 217–222, Feb. 2005.

[30] H. B. Zhang, C. G. Li, and X. F. Liao, "Stability analysis and H_∞ controller design of fuzzy large-scale systems based on piecewise Lyapunov functions," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 36, no. 3, pp. 685–698, Jun. 2006.

[31] H. G. Zhang, S. X. Lun, and D. Liu, "Fuzzy H_∞ filter design for a class of nonlinear discrete-time systems with multiple time delay," *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 3, pp. 453–469, Jun. 2007.

[32] H. G. Zhang and Y. C. Wang, "Delay-dependent guaranteed cost control for uncertain stochastic fuzzy systems with time delays," *Prog. Nat. Sci.*, vol. 17, no. 1, pp. 95–101, Jan. 2007.

Huaguang Zhang (SM'04) was born in Jilin, China, in 1959. He received the B.S. and M.S. degrees in control engineering from the Northeastern Electric Power University of China, Jilin, in 1982 and 1985, respectively, and the Ph.D. degree in thermal power engineering and automation from Southeast University, Nanjing, China, in 1991.

In 1992, he joined the Department of Automatic Control, Northeastern University, Shenyang, China, as a Postdoctoral Fellow for two years. Since 1994, he has been a Professor and Head of the Institute of Electric Automation, Northeastern University. He is also with the Key Laboratory of Integrated Automation of Process Industry (Northeastern University), National Education Ministry, Shenyang. His research interests are fuzzy control, stochastic system control, neural-networks-based control, nonlinear control, and their applications.

Dr. Zhang is currently an Associate Editor for the IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS—PART B. He was the recipient of the Outstanding Youth Science Foundation Award from the National Natural Science Foundation of China in 2003 and was named the Cheung Kong Scholar by the China Education Ministry in 2005.

Yingchun Wang was born in Liaoning Province, China, in 1974. He received the B.S., M.S., and Ph.D. degrees from Northeastern University, Shenyang, China, in 1997, 2003, and 2006, respectively.

Since 2006, he has been with the School of Information Science and Engineering, Northeastern University. He is also currently with the Key Laboratory of Integrated Automation of Process Industry (Northeastern University), National Education Ministry, Shenyang. His research interests include fuzzy control and fuzzy systems, stochastic control, time-delay systems, and nonlinear systems.

Derong Liu (S'91–M'94–SM'96–F'05) received the B.S. degree in mechanical engineering from the East China Institute of Technology (now Nanjing University of Science and Technology), Nanjing, in 1982, the M.S. degree in electrical engineering from the Chinese Academy of Sciences (CAS), Beijing, in 1987, and the Ph.D. degree in electrical engineering from the University of Notre Dame, Notre Dame, IN, in 1994.

From 1982 to 1984, he was a Product Design Engineer with China North Industries Corporation, Jilin, China. From 1987 to 1990, he was an Instructor with the Graduate School of CAS. From 1993 to 1995, he was a Staff Fellow with the General Motors Research and Development Center, Warren, MI. From 1995 to 1999, he was an Assistant Professor with the Department of Electrical and Computer Engineering, Stevens Institute of Technology, Hoboken, NJ. In 1999, he joined the University of Illinois, Chicago, where he is currently a Full Professor of electrical and computer engineering and of computer science. Since 2005, he has been the Director of Graduate Studies with the Department of Electrical and Computer Engineering, University of Illinois. He is a coauthor of *Dynamical Systems With Saturation Nonlinearities: Analysis and Design* (Springer-Verlag, 1994) and *Qualitative Analysis and Synthesis of Recurrent Neural Networks* (Marcel Dekker, 2002), and *Fuzzy Modeling and Fuzzy Control* (Birkhauser, 2006). He is a Coeditor of *Stability and Control of Dynamical Systems With Applications* (Birkhauser, 2003), *Advances in Computational Intelligence: Theory and Applications* (World Scientific, 2006), and *Advances in Neural Networks-ISNN2007* (Springer-Verlag, 2007). He is also an Associate Editor of *Automatica*.

Dr. Liu has been an Associate Editor for the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—PART I: FUNDAMENTAL THEORY AND APPLICATIONS (1997–1999), the IEEE TRANSACTIONS ON SIGNAL PROCESSING (2001–2003), and the IEEE TRANSACTIONS ON NEURAL NETWORKS (2004–2006). Since 2004, he has been the Editor for the IEEE Computational Intelligence Society's ELECTRONIC LETTER, and since 2006, he has been the Letter Editor for the IEEE TRANSACTIONS ON NEURAL NETWORKS and an Associate Editor for the IEEE COMPUTATIONAL INTELLIGENCE MAGAZINE. He is the General Chair for the 2008 IEEE International Conference on Networking, Sensing and Control. He was the General Chair for the 2007 International Symposium on Neural Networks (Nanjing, China). He is the Program Chair for the 2008 International Joint Conference on Neural Networks; the 2007 IEEE International Symposium on Approximate Dynamic Programming and Reinforcement Learning; the 21st IEEE International Symposium on Intelligent Control (2006); and the 2006 International Conference on Networking, Sensing and Control. He has served and is serving as a member of the organizing committee and the program committee of several other international conferences. He is an elected AdCom Member of the IEEE Computational Intelligence Society (2006–2008), Chair of the Chicago Chapter of the IEEE Computational Intelligence Society, and Past Chair of the Technical Committee on Neural Systems and Applications of the IEEE Circuits and Systems Society. He was the recipient of the Michael J. Birck Fellowship from the University of Notre Dame (1990), the Harvey N. Davis Distinguished Teaching Award from the Stevens Institute of Technology (1997), the Faculty Early Career Development (CAREER) Award from the National Science Foundation (1999), and the University Scholar Award from the University of Illinois (2006–2009). He is a Member of Eta Kappa Nu and the Conference Editorial Board of the IEEE Control Systems Society (1995–2000).