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CHAOTIFYING FUZZY HYPERBOLIC MODEL USING IMPULSIVE AND NONLINEAR FEEDBACK CONTROL APPROACHES*

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In this paper, the problem of chaotifying the continuous-time fuzzy hyperbolic model (FHM) is considered. We use impulsive and nonlinear feedback control methods to chaotify the FHM and show that chaos produced by the present methods satisfy the three criteria of Devaney. Computer simulations will be used to verify the present results.

Keywords: Chaotification; Devaney's chaos; fuzzy hyperbolic model; impulsive control; Poincaré section.

1. Introduction

In the past decade, there have been some studies about control methods with the purpose of either reducing "bad" chaos or introducing "good" chaos [Chen & Dong, 1998]. Due to its great potential in nontraditional applications such as those found within the context of physical, chemical, mechanical, electrical, optical, and particularly, biological and medical systems [Chen *et al.*, 2001; Schiff *et al.*, 1994; Yang *et al.*, 1995], making a nonchaotic system chaotic or maintaining existing chaos, known as "chaotification" or "anticontrol", has attracted increasing attention in recent years. The process of chaos control is now understood as a transition from chaos to order and sometimes from order to chaos, depending on the purposes of different applications.

Studies have shown that discrete maps can be chaotified in the sense of Devaney or Li–Yorke by a state-feedback controller with a uniformly bounded control-gain sequence designed to make all Lyapunov exponents of the controlled system strictly positive or arbitrarily assigned [Chen & Lai, 1997; Chen & Lai, 1998; Wang & Chen, 1999; Wang & Chen, 2000a, 2000b, 2000c]. Even though there is some research work indicating that a kind of continuous stable systems can be chaotified [Lü *et al.*, 2002; Sanchez *et al.*, 2001; Tang *et al.*, 2001; Wang *et al.*, 2000; Wang *et al.*, 2001; Yang *et al.*, 2002; Zhong *et al.*, 2001], it is still an open problem on

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how to chaotify general nonlinear systems that cannot be linearized.

Fuzzy control methods have been widely used in controlling chaotic systems. Successful applications have been demonstrated, in particular, in situations where the dynamics of systems are so complex that it is impossible to construct an accurate model [Chen & Chen, 1999; Tanaka *et al.*, 1998]. The present work makes contributions to the growing literature on the chaotification of nonchaotic fuzzy systems. First, such a study is of academic interest. In general, a fuzzy model is a nonlinear system and it is difficult to chaotify. Second, it is well known that fuzzy systems with universal approximation property can display all kinds of dynamics when there are enough rules [Passino & Yurkovich, 1998]. Based on this fact, we can consider the problem of chaotifying a continuous-time fuzzy model instead of the problem of chaotifying the original system when this fuzzy model describes the system well enough in the real world. If such a fuzzy model can be chaotified by a controller, it is reasonable to believe that the real system can also be chaotified by the same controller. To chaotify a fuzzy model, the common method is to model a chaotic system with enough rules. It is natural to ask whether a fuzzy system with a fixed number of rules can be chaotified by control inputs. If it can, then the question becomes how to design such a controller. Part of the answer has been given in [Li *et al.*, 2003] where the continuous-time Takagi–Sugeno (T–S) fuzzy model was chaotified under certain conditions. Parallel to the T–S fuzzy model, a new continuous-time fuzzy model, called the fuzzy hyperbolic model (FHM), has been proposed recently [Zhang & Quan, 2001]. There are many merits compared to the T–S fuzzy model when modeling a nonlinear system using FHM. For example, because there is no need to identify premise structure at the modeling phase, the computation using FHM is less than that of using T–S fuzzy model. Besides, an FHM can also be regarded as a neural network. Therefore, one can use neural network learning methods to tune parameters [Zhang & Quan, 2001; Zhang *et al.*, 2004a]. Meanwhile, an FHM is a special case of Lur'e systems, and a number of research work have been presented on how to synchronize such systems [Suykens *et al.*, 1997a, 1997b, 1998, 1999]. The chaotification of the T–S model using adaptive inverse optimal control approach has been studied in [Zhang *et al.*, 2004b]. In this paper, the focus is on how to make an FHM chaotified. Using the impulsive control technique which has been successfully used for the control and synchronization of chaotic systems [Guan *et al.*, 2000; Li *et al.*, 2001; Suykens *et al.*, 1998; Yang & Chua, 1997, 2000; Yuan *et al.*, 2002] and nonlinear feedback control, we have succeeded in chaotifying the FHM. We also provide a proof that a controlled FHM can produce chaos, satisfying the Devaney's definition. The present method can also be extended to a wide class of nonlinear systems.

This paper is organized as follows. In Sec. 2, preliminaries are reviewed for the Devaney's chaos and the FHM. In Sec. 3, the chaotification problem considered in this paper is described. In Sec. 4, a new method that can chaotify the FHM is provided. In Sec. 5, simulation results are presented, and in Sec. 6, conclusions are given.

2. Preliminaries

In this section we review some necessary preliminaries for Devaney's chaos and the FHM.

Definition 1 [Devaney, 1987]. A map $\phi : S \to S$, where S is a set, is chaotic if

(i) ϕ has sensitive dependence on initial conditions, i.e. for any $x \in S$ and any neighborhood N of x in S, there exists a $\delta > 0$ such that $\|\phi^m(x) - \phi^m(y)\| > \delta$ for some $y \in N$ and $m > 0$, where ϕ^m is the mth-order iteration of ϕ, i.e. $\phi^m := \phi \circ \phi \circ \cdots \circ \phi$ (m times).
(ii) ϕ is topological transitive, i.e. for any pair of subsets $U, V \subset S$, there exists an integer $m > 0$ such that $\phi^m(U) \cap V \neq \emptyset$.
(iii) The periodic points of ϕ are dense in S.

Remark 1. Definition 1 is for discrete-time systems. For continuous-time systems, we need to construct a Poincaré section to get a Poincaré map. Details will be provided later.

Definition 2. Given a plant with n input variables $x = (x_1(t), \ldots, x_n(t))^T$ and an output variable $\dot{x}$, we call the fuzzy rule base "hyperbolic type fuzzy rule base" if it satisfies the following conditions:

(i) Every fuzzy rule has the following form:

$$R^l: \text{ IF } x_1 \text{ is } F_{x_1}, x_2 \text{ is } F_{x_2}, \ldots, \text{and } x_n \text{ is } F_{x_n}, \\ \text{THEN } \dot{x} = \pm c_{x_1} \pm c_{x_2} \pm \cdots \pm c_{x_n}, \\ l = 1, \ldots, 2^n,$$

where F_{x_i} $(i = 1, \ldots, n)$ are fuzzy sets of x_i, which include P (positive) and N (negative), c_{x_i} $(i = 1, \ldots, n)$ are positive constants corresponding to F_{x_i}, and $\pm$ stands for either the plus or the minus sign. The actual signs in the THEN-part are determined in the following manner: If in the IF-part the term characterizing F_{x_i} is P, then in the THEN-part c_{x_i} appears with a plus sign; otherwise, c_{x_i} appears with a minus sign.

(ii) The constant terms c_{x_i} in the THEN-part correspond to F_{x_i} in the IF-part; that is, if there is an F_{x_i} term in the IF-part, c_{x_i} must appear in the THEN-part. Otherwise, c_{x_i} does not appear in the THEN-part.

(iii) There are 2^n fuzzy rules in the rule base; that is, all the possible P and N combinations of input variables in the IF-part, and all the sign combinations of constants in the THEN-part.

The following theorem explains how an FHM is constructed.

Theorem 1 [Zhang & Quan, 2001]. *Given a hyperbolic type rule base, if we define the membership function of P_{x_i} and N_{x_i} as*:

$$\mu_{P_{x_i}}(x_i) = e^{-\frac{1}{2}(x_i - k_{x_i})^2}, \quad \mu_{N_{x_i}}(x_i) = e^{-\frac{1}{2}(x_i + k_{x_i})^2}$$

where $k_{x_i} > 0$, then we can always derive the following model:

$$\dot{x} = A \tanh(Kx) \tag{1}$$

where $K = \mathrm{diag}[k_{x_1}, \ldots, k_{x_n}]$ and A is a constant matrix.

Definition 3 [Kaszkurewicz & Bhaya, 2000]. A matrix A is said to be a Hurwitz diagonally stable matrix if there exits a diagonal matrix $Q > 0$ such that $A^T Q + QA < 0$.

Lemma 1 [Kaszkurewicz & Bhaya, 2000]. *For system (1), if A is a Hurwitz diagonally stable matrix, then $x = 0$ is globally asymptotically stable.*

3. Main Results

Consider the following control system:

$$\dot{x} = Af(x) + Bu \tag{2}$$

where $x \in R^n$, $f(x) = \tanh(Kx)$ is an n-dimension vector function, $u = u(x(t)) \in R^m$ is a vector function of states x, and $A \in R^{n \times n}$ and $B \in R^{n \times m}$ are both constant matrices.

Assumption 1. (A, B) is completely controllable.

Under this assumption, we have the following lemma which is a direct consequence of Lemma 1.

Lemma 2. *For system (2), if the controller u is chosen as follows*:

$$u(x(t)) = L \tanh(Kx(t)), \tag{3}$$

where $L \in R^{m \times n}$ is a constant matrix such that $(A + BL)$ is a Hurwitz diagonally stable matrix, then the origin $x = 0$ of the closed-loop system (2) is globally asymptotically stable.

Remark 2. For the purpose of deriving our main results of the present paper, we only require the stability of closed-loop system (2) and (3). It is noticed that there are other results for the stability of closed-loop system (2) and (3) which can also be applied here. System (1) and the closed-loop system (2) and (3) can be regarded as recurrent neural networks. They are also special cases of Lur'e systems [Khalil, 1996].

Remark 3. Under the conditions of Lemma 2, solutions of the control system (2) are defined in a bounded region $D \subset R^n$.

Remark 4. If we let $\tilde{A} = A + BL$, system (2) with u given in (3) becomes

$$\dot{x} = \tilde{A} f(x). \tag{4}$$

Because of the form of $f(x)$, there exits a constant γ such that for all different $x, y \in D$,

$$\|f(y) - f(x)\| \le \gamma \|y - x\|,$$

that is to say, $f(x)$ is Lipschitz over D. From the theory of ordinary differential equations, we know that system (2) has a unique continuous solution $\phi(t, t_0, x_0)$ through a given initial point, (t_0, x_0), which is also continuously dependent on x_0 [Hirsch & Smale, 1974].

Now, let us consider the following impulsive control system:

$$\begin{aligned} \dot{x} &= \tilde{A} f(x), \quad t \ne \tau_k \\ \Delta x &= I_k(x(t)), \quad t = \tau_k, \ k = 1, 2, 3, \ldots \\ x(t_0+) &= x_0, \quad t_0 \ge 0 \end{aligned} \tag{5}$$

where $\tilde{A}$ is defined in (4), $I_k(x(t))$, $k = 1, 2, 3, \ldots$, are impulsive control signals defined in $D \subset R^n$, $t_0 = \tau_0 < \tau_1 < \cdots < \tau_k$, $\tau_k - \tau_{k-1} = T$, $\lim \tau_{k\, k \to +\infty} = +\infty$, $\Delta x = x(\tau_k+) - x(\tau_k-)$, and

$x(\tau_k+)$ and $x(\tau_k-)$ are the right and left limits of $x(t)$ at $t = \tau_k$.

For system (5), we have the following lemma:

Lemma 3. *If $I_k(x(t))$ is chosen as follows*:

$$I_k(x(t)) = y_k - x(t-), \quad t = \tau_k, \ k = 1, 2, 3, \ldots \tag{6}$$

where $y_k = g(y_{k-1})$ and $g: Y \to Y$, $Y \subset D$. Then, when $g(\cdot)$ is chaotic and satisfies the three criteria of Devaney, the control system (5) is also chaotic and satisfies the Devaney's criteria.

Proof. First, it should be emphasized that for system (5) to display chaotic dynamics, its phase space must be a finite region; that is to say, there exists an $M > 0$ such that the trajectory of system (5), $\phi(t, t_0, x_0)$, satisfies $\|\phi(t, t_0, x_0)\| \le M$ for all t in the domain. In fact, for $t \in (\tau_0, \tau_1)$, system (5) is under the action of $\tilde{A}f(x)$. By Lemma 2, its trajectory, $\phi(t, t_0+, x_0)$, is asymptotically stable. When $t = \tau_1$, $\tilde{A}f(x)$ turns off and $I_1(x(\tau_1))$ acts on system (5). The trajectory of system (5) jumps from $\phi(\tau_1-, t_0+, x_0)$ to x_0^1. We denote the trajectory at each instant $t = \tau_k$ as x_0^k, $k = 0, 1, 2, \ldots$, where $x_0^0 = x_0$. Since $I_1(x(\tau_1))$ is a finite impulse, x_0^1 is also finite. For $t \in (\tau_1, \tau_2)$, $I_1(x(\tau_1))$ turns off and $\tilde{A}f(x)$ turns on with initial point (τ_1+, x_0^1). The trajectory of system (5) in this time span, denoted by $\phi(t, \tau_1+, x_0^1)$, is also asymptotically stable. The analysis in other time spans, (τ_k, τ_{k+1}), $k = 2, 3, 4, \ldots$, are the same as that in (τ_1, τ_2), and situations at other time instants $t = \tau_k$, $k = 2, 3, 4, \ldots$, are analogous to that at $t = \tau_1$. It is easy to see that although the trajectory may be discontinuous at $t = \tau_k$, $k = 1, 2, 3, \ldots$, the whole trajectory is indeed in a bounded region.

In the following we will prove that system (5) is chaotic and satisfies the Devaney's three criteria.

To prove that a continuous-time system is chaotic, one method is to show that its Poincaré map is chaotic [Guckenheimer & Holmes, 1983]. Suppose that $\phi(t, t_0+, x_0)$ is a solution of system (4) with initial value $x(t_0+) = x_0$. According to Remark 4, the solution $\phi(t, t_0+, x_0)$ is unique and thus, $\phi^{-1}(t, t_0+, x_0)$ exists. Define a set of Poincaré sections S_k as

$$S_k = \{(t, x) | x \in D, t = \tau_k\}, \quad k = 1, 2, 3, \ldots.$$

The Poincaré map is defined as:

$$P: V \to V, \quad P = \psi \circ g \circ \psi^{-1},$$

where $V = \psi(Y)$ and

$$\psi(x) = \phi(\tau_k + T, \tau_k, x) = \phi(\tau_0 + T, \tau_0, x).$$

We will first prove that P is extremely sensitive to initial values, i.e. there exists an $\varepsilon > 0$ and for two points $x_0, \overline{x}_0 \in V$, there exists an $N > 0$, such that

$$\|P^N(x_0) - P^N(\overline{x}_0)\| > \varepsilon. \tag{7}$$

Since

$$\begin{aligned} P^N &= \underbrace{(\psi \circ g \circ \psi^{-1}) \circ \cdots \circ (\psi \circ g \circ \psi^{-1})}_{N\text{th-order iteration of } P} \\ &= \psi \circ g^N \circ \psi^{-1}, \end{aligned} \tag{8}$$

to prove that (7) holds is equivalent to prove that

$$\|\psi(g^N(y_0)) - \psi(g^N(\overline{y}_0))\| > \varepsilon \tag{9}$$

holds, where $y_0 = \psi^{-1}(x_0)$, $\overline{y}_0 = \psi^{-1}(\overline{x}_0)$.

If (9) does not hold, i.e. for all $n \ge 0$, there exists an $\varepsilon_0 > 0$ such that

$$\|\psi(g^n(y_0)) - \psi(g^n(\overline{y}_0))\| \le \varepsilon_0.$$

Thus, from the fact that $\psi(x)$ is continuous we know that for the above ε_0, there exists a δ_0 such that

$$\|g^n(y_0) - g^n(\overline{y}_0)\| < \delta_0 \quad \text{for all } n \ge 0.$$

But this contradicts the fact that g is a chaotic map satisfying the Devaney's definition. Therefore, P satisfies the first criterion of Devaney.

Next, we show that P is topologically transitive; that is, for any pair of subsets $E, F \subset V$, there exists an integer $N > 0$ such that

$$P^N(E) \cap F \ne \emptyset. \tag{10}$$

It is known that for any two subsets $\psi^{-1}(E)$ and $\psi^{-1}(F) \subset Y$, there exists a number $N > 0$ such that

$$g^N[\psi^{-1}(E)] \cap \psi^{-1}(F) \ne \emptyset. \tag{11}$$

Acting on both sides of (11) by ψ, we get

$$\psi\{g^N[\psi^{-1}(E)]\} \cap F \ne \emptyset.$$

Noticing (8), we therefore proved (10).

Finally, we will prove that the periodic points of P are dense in V. Denote the set of periodic points of a map f as $\overline{\mathrm{Per}(f)}$. Since $\overline{\mathrm{Per}(g)} = Y$ and ψ is one to one, we have $\overline{\mathrm{Per}(P)} = V$.

Thus, we have proved the lemma. ■

Summarizing the results above, we have the following theorem:

Theorem 2. *For the following system*

$$\begin{gathered} \dot{x} = Af(x) + Bu, \quad t \neq \tau_k \\ \Delta x = I_k(x(t)), \quad t = \tau_k, \ k = 1, 2, 3, \ldots \\ x(t_0+) = x_0, \quad t_0 \geq 0 \end{gathered} \tag{12}$$

if u is chosen as in (3) and $I_k(x(t))$ is chosen as in (6), the system (12) will display chaotic dynamics and satisfies the three criteria of Devaney.

Remark 5. The conclusion of Theorem 2 can also be kept if the impulsive control is chosen as $I_k(x(t)) = h(y_k) - x(t-)$, where $h: Y \to Y$ is a homeomorphism. A special case is to choose $h(y_k) = \Lambda y_k$ with Λ a constant diagonal matrix $\Lambda = \mathrm{diag}[\lambda_1, \ldots, \lambda_n]$ with $\lambda_i > 0, i = 1, \ldots, n$, which means that we can chaotify the original system with arbitrarily small impulsive energy.

4. Simulations

Suppose that we have the following fuzzy rule base:

If x_1 is P_{x1} and x_2 is P_{x2}, then $\dot{x}_3 = C_{x1} + C_{x2}$;
If x_1 is N_{x1} and x_2 is P_{x2}, then $\dot{x}_3 = -C_{x1} + C_{x2}$;
If x_1 is P_{x1} and x_2 is N_{x2}, then $\dot{x}_3 = C_{x1} - C_{x2}$;
If x_1 is N_{x1} and x_2 is N_{x2}, then $\dot{x}_3 = -C_{x1} - C_{x2}$;
If x_1 is P_{x1} and x_3 is P_{x3}, then $\dot{x}_2 = C_{x1} + C_{x3}$;
If x_1 is N_{x1} and x_3 is P_{x3}, then $\dot{x}_2 = -C_{x1} + C_{x3}$;
If x_1 is P_{x1} and x_3 is N_{x3}, then $\dot{x}_2 = C_{x1} - C_{x3}$;
If x_1 is N_{x1} and x_3 is N_{x3}, then $\dot{x}_2 = -C_{x1} - C_{x3}$;
If x_2 is P_{x2} and x_3 is P_{x3}, then $\dot{x}_1 = C_{x2} + C_{x3}$;
If x_2 is N_{x2} and x_3 is P_{x3}, then $\dot{x}_1 = -C_{x2} + C_{x3}$;
If x_2 is P_{x2} and x_3 is N_{x3}, then $\dot{x}_1 = C_{x2} - C_{x3}$;
If x_2 is N_{x2} and x_3 is N_{x3}, then $\dot{x}_1 = -C_{x2} - C_{x3}$.

Here, we choose membership functions of P_{xi} and N_{xi} as follows:

$$\mu_{P_{xi}}(x) = e^{-\frac{1}{2}(x_i - k_i)^2}, \quad \mu_{N_{xi}}(x) = e^{-\frac{1}{2}(x_i + k_i)^2}.$$

Then we have the following third-order model:

$$\dot{x} = Af(x) = A \tanh(Kx)$$

where $x = [x_1 \ x_2 \ x_3]^T$, $A = \begin{bmatrix} 0 & C_{x2} & C_{x3} \\ C_{x1} & 0 & C_{x3} \\ C_{x1} & C_{x2} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 3 \\ 2 & 0 & 3 \\ 2 & 1 & 0 \end{bmatrix}$ and $K = \mathrm{diag}[k_1 \ k_2 \ k_3] = \mathrm{diag}[2 \ 3 \ 1]$. According to Theorem 2, the control system is

$$\begin{gathered} \dot{x} = (A + BL)f(x), \quad t \neq \tau_k \\ \Delta x = I_k(x(t)), \quad t = \tau_k, \ k = 1, 2, 3, \ldots \\ x(t_0+) = x_0. \end{gathered} \tag{13}$$

We select $B = [1 \quad 1 \quad 1]^T$, $L = [-2 \quad -1 \quad -2]$ and $Q = \mathrm{diag}[q_1 \quad q_2 \quad q_3] = \mathrm{diag}[2 \quad 2 \quad 2]$. It is easy to verify that matrix $A + BL$ is Hurwitz diagonally stable. The initial value is $x_0 = [5 \quad 2 \quad -1]^T$ and the impulsive control is

$$I_k(x) = \Lambda g(y_{k-1}) - x(t-),$$

where $\Lambda = \mathrm{diag}[\lambda_1 \quad \lambda_2 \quad \lambda_3]$ and $g(y_{k-1}) = [g_1(y_{k-1}^1) \quad g_2(y_{k-1}^2) \quad g_3(y_{k-1}^3)]^T$ with $g_i(\cdot)$ a logistic map:

$$y_k^i = g_i(y_{k-1}^i) = a y_{k-1}^i (1 - y_{k-1}^i), \quad i = 1, 2, 3.$$

When $a = 4.0$, the logistical map is chaotic; see Fig. 1.

Without impulsive control, the following system

$$\dot{x} = (A + BL)f(x) \tag{14}$$

is globally asymptotically stable, its trajectories are shown in Fig. 2.

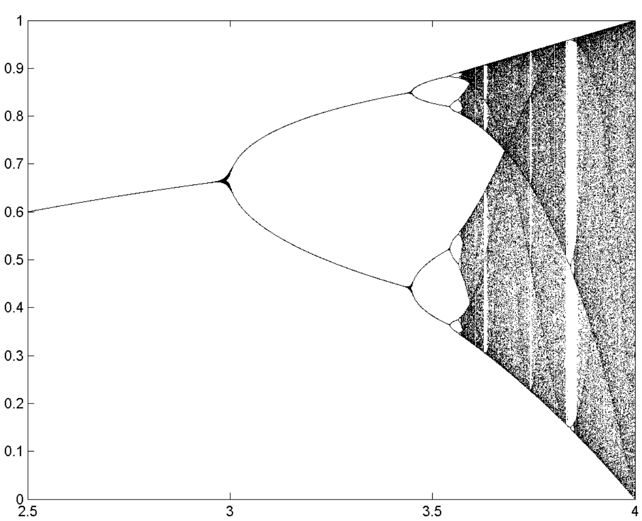
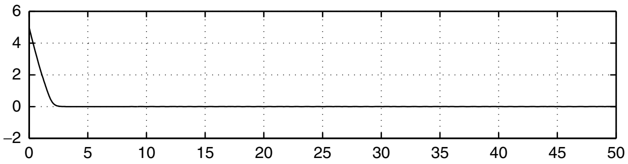
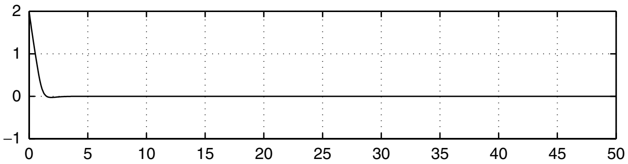


Fig. 1. The bifurcation diagram of logistic map.



(a)



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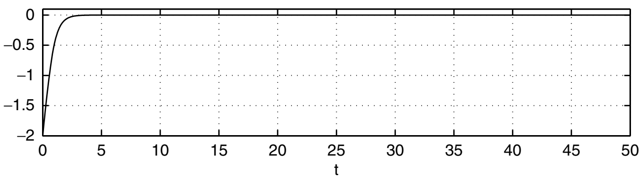
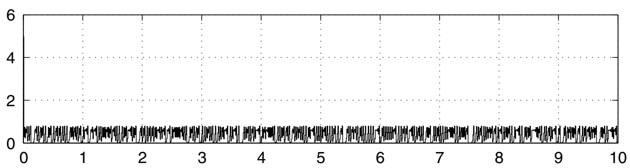


(c)

Fig. 2. State curves of (14). The state curve of (a) x_1; (b) x_2; (c) x_3.



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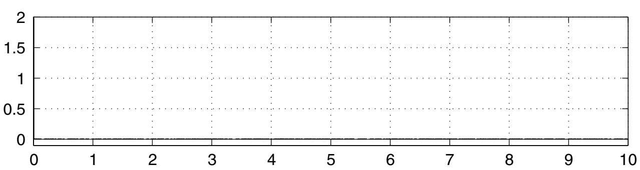

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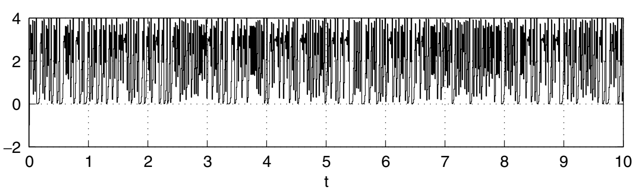


(c)

Fig. 3. State curves of system (13) when $\Lambda = [0.8 \quad 0 \quad 4]$. The curve of (a) x_1; (b) x_2; (c) x_3.

With impulsive controls and $T = 0.01$, from Figs. 3 to 6, we can see that the control system is in chaos for different Λ. These results verify the claims of Theorem 2 and Remark 5.

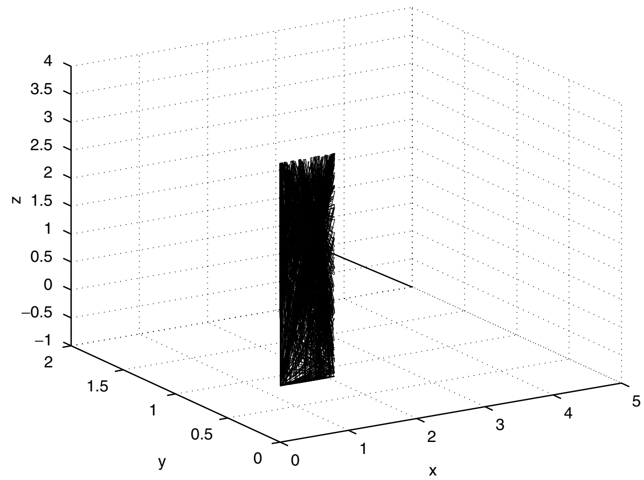
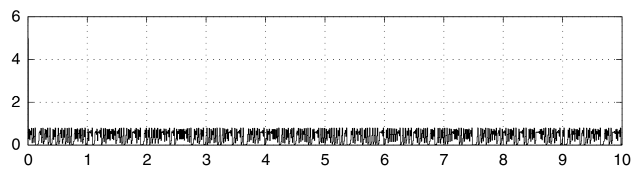
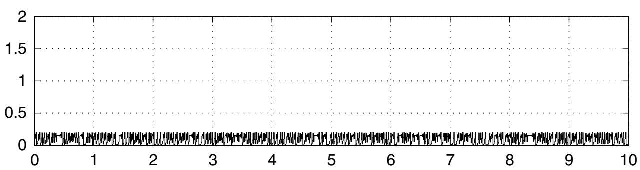


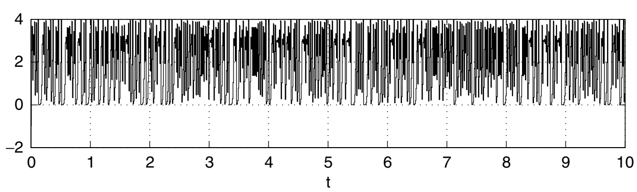
Fig. 4. Phase diagram of system (13) when $\Lambda = [0.8 \quad 0 \quad 4]$.



(a)



(b)



(c)

Fig. 5. State curves of system (13) when $\Lambda = [0.8 \quad 0.2 \quad 4]$. The curve of (a) x_1; (b) x_2; (c) x_3.

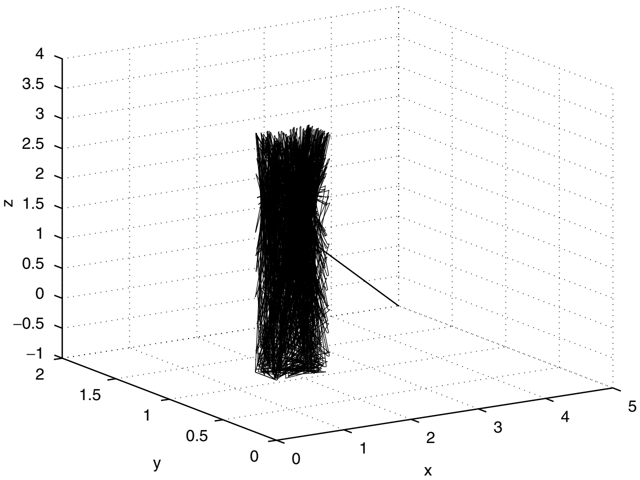


Fig. 6. Phase diagram of system (13) when $\Lambda = [0.8 \quad 0.2 \quad 4]$.

5. Conclusions

In this paper, the problem of chaotifying the fuzzy hyperbolic model (FHM) by nonlinear state feedback and impulsive control is investigated. A proof is given that the FHM can be chaotified using the method developed in this paper. Computer simulations also verify the method developed in this paper. We believe that this anticontrol method can be used not only for the FHM, but also for a wide class of nonchaotic systems.

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