Fuzzy H_∞ Filter Design for a Class of Nonlinear Discrete-Time Systems With Multiple Time Delays

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***Abstract*—This paper studies the fuzzy H_∞ filter design problem for signal estimation of nonlinear discrete-time systems with multiple time delays and unknown bounded disturbances. First, the Takagi–Sugeno (T–S) fuzzy model is used to represent the state-space model of nonlinear discrete-time systems with time delays. Next, we design a stable fuzzy H_∞ filter based on the T–S fuzzy model, which guarantees asymptotic stability and a prescribed H_∞ index for the filtering error system, irrespective of the time delays and uncertain disturbances. A sufficient condition for the existence of such a filter is established by using the linear matrix inequality (LMI) approach. The proposed LMI problem can be efficiently solved with global convergence guarantee using convex optimization techniques such as the interior point algorithm. Simulation examples are provided to illustrate the design procedure of the present method.**

***Index Terms*—Discrete-time systems, H_∞ filtering, linear matrix inequality, Lyapunov function, Takagi–Sugeno fuzzy model, time delays.**

I. INTRODUCTION

RECENTLY, there has been a lot of interest in the problem of robust H_∞ filtering of systems with uncertain external disturbances and measurement noises [5], [16], [19]. The advantage of using an H_∞ filter over a Kalman filter is that no statistical assumption on the noise signals is needed. In robust H_∞ filtering, the noise signals are assumed to be arbitrary signals with bounded energy (or average power). The H_∞ filter is designed by minimizing signal estimation error for the bounded disturbances and noises of the worst case. Thus, the H_∞ filter is more robust than the Kalman filter. Moreover, the H_∞ filtering approach provides both a guaranteed noise attenuation level and a strong robustness against unmodeled dynamics.

Several robust H_∞ filtering approaches for linear systems have been developed over the past few years [8], [9], [30], [35]. However, it is difficult to design an efficient filter for signal estimation in nonlinear systems [3]. To the best of our knowledge, the problem of H_∞ filtering of nonlinear discrete-time state-space models with multiple time delays has not been fully investigated in the literature. As is well known, time delays usually exist in many dynamic systems. Since time delays usually result in unsatisfactory performance and are frequently a source of instability, their presence must be taken into account in practical filter designs [14]. It turns out that the noise attenuation level guaranteed by an H_∞ filter design without considering nonnegligible time delays may become invalid in the presence of time delays. Time delays often arise in many signal-processing related problems, such as echo cancellation, local loop equalization, multipath propagation in mobile communications, and array signal processing [2], [11], [15]. Several robust H_∞ filtering approaches for linear systems with multiple time delays have been developed over the past few years [4], [7], [17], [18], [27], [31].

This paper deals with the fuzzy H_∞ filter design problem for nonlinear discrete-time systems with time delays in the state variables. Recently, there have been many successful applications of fuzzy systems theory in filtering and control applications [6], [10], [13], [29]. The Takagi–Sugeno (T–S) fuzzy systems are considered to be universal approximators for certain nonlinear systems and they have been proved to be a good representation for a class of nonlinear dynamical systems [12], [20], [25], [28], [33], [34]. In [24], for nonlinear discrete-time systems, an H_∞ filter based on T–S fuzzy model is developed. The operator methodology of functional analysis is used to analyze and design an fuzzy H_∞ filter. H_∞ performance index is used directly to derive the existence condition for H_∞ filters. However, the time-delay problem was not considered, which limited the application of the results of [24]. The design of an H_∞ filter requires solving LMIs with a common positive definite matrix P that has a special block diagonal structure [24], which leads to conservative designs of fuzzy filters. In this paper, our result requires a less conservative condition for the analysis and design of fuzzy H_∞ filters. Moreover, our analysis and design are based on the Lyapunov stability theory and thus stability is always guaranteed for filters designed using the present approach. In addition, this paper can also be used to design filters for systems with time delays.

In this paper, first, the T–S fuzzy model is proposed to interpolate a class of nonlinear systems with time delays. The fuzzy H_∞ filter design problem based on the T–S fuzzy model will be addressed next, which guarantees asymptotic stability and a prescribed H_∞ index for the filtering error system, irrespective of the time delays and the uncertainties. Finally, the fuzzy H_∞

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filter design problem is converted into a linear matrix inequality (LMI) problem, which makes the prescribed attenuation level as small as possible, subject to some LMI constraints. The LMI problem can efficiently be solved by the convex optimization techniques with global convergence, such as the interior point algorithm [1].

This paper is organized as follows. In Section II, the modeling of a class of nonlinear discrete-time systems with multiple time delays is addressed using the T–S fuzzy system. In Section III, an analysis is presented for fuzzy H_∞ filtering based on the T–S fuzzy model of nonlinear discrete-time systems with time delays. In Section IV, the fuzzy H_∞ filter design based on the T–S fuzzy model is addressed. In Section V, a simulation example is provided to demonstrate the design procedure of fuzzy H_∞ filters. Finally, in Section VI, concluding remarks are given.

II. The Modeling of a Class of Nonlinear Discrete-Time Systems With Time Delays Using the T–S Fuzzy System

Consider a class of nonlinear discrete-time systems with multiple time delays

$$\begin{aligned}\mathbf{x}(k+1) &= F(\mathbf{x}(k)) + F_d(\mathbf{x}(k-d_1), \mathbf{x}(k-d_2), \dots, \\ &\qquad \mathbf{x}(k-d_q)) + B\mathbf{w}(k) \\ \mathbf{y}(k) &= H(\mathbf{x}(k)) + H_d(\mathbf{x}(k-d_1), \mathbf{x}(k-d_2), \dots, \\ &\qquad \mathbf{x}(k-d_q)) + G\mathbf{v}(k) \\ \mathbf{s}(k) &= L\mathbf{x}(k) + \sum_{j=1}^{q} L_{dj}\mathbf{x}(k-d_j) \end{aligned} \tag{1}$$

where $\mathbf{x}(k) = [x_1(k), x_2(k), \dots, x_n(k)]^T \in R^{n\times 1}$ denotes the state vector; $\mathbf{y}(k) \in R^{m\times 1}$ denotes the measurements vector; $\mathbf{s}(k) \in R^{p\times 1}$ denotes the signal vector to be estimated; $\mathbf{w}(k) \in R^{n\times 1}$ and $\mathbf{v}(k) \in R^{m\times 1}$ are assumed to be bounded external disturbance vector and measurement noise vector, respectively; $d_1, d_2, \dots, d_q$ denote bounded time delays of the state; $B \in R^{n\times n}$, $G \in R^{m\times m}$, $L \in R^{p\times n}$, and $L_{dj} \in R^{p\times n}$ are constant matrices; and $F(\mathbf{x}(k))$ and $F_d(\mathbf{x}(k-d_1), \mathbf{x}(k-d_2), \dots, \mathbf{x}(k-d_q)) \in R^{n\times 1}$, $H(\mathbf{x}(k)), H_d(\mathbf{x}(k-d_1), \mathbf{x}(k-d_2), \dots, \mathbf{x}(k-d_q)) \in R^{m\times 1}$ are vector fields with $F(0) = 0$, $H(0) = 0$, $F_d(0, 0, \dots, 0) = 0$, and $H_d(0, 0, \dots, 0) = 0$. The initial conditions are given by $\mathbf{x}(k) = 0$ for $k = -d_{\max}, -d_{\max}+1, \dots, -1$ and $\mathbf{x}(0) = \mathbf{x}_0$, where $d_{\max} = \max_{1\le j\le q}(d_j)$. For the sake of convenience, define $\mathbf{d} = (d_1, d_2, \dots, d_q)$. We denote

$$\begin{aligned}&[\mathbf{x}^T(k-d_1), \mathbf{x}^T(k-d_2), \dots, \mathbf{x}^T(k-d_q)]^T \\ &\quad \triangleq \mathbf{x}(k, \mathbf{d}, q) \\ &F_d(\mathbf{x}(k-d_1), \mathbf{x}(k-d_2), \dots, \mathbf{x}(k-d_q)) \\ &\quad \triangleq F_d(\mathbf{x}(k, \mathbf{d}, q)) \\ &H_d(\mathbf{x}(k-d_1), \mathbf{x}(k-d_2), \dots, \mathbf{x}(k-d_q)) \\ &\quad \triangleq H_d(\mathbf{x}(k, \mathbf{d}, q)).\end{aligned}$$

The ith rule of the T–S fuzzy model for the nonlinear discrete-time systems with multiple time delays in (1) is given as follows.

Rule i: IF $x_1(k)$ is $F_{i1}, \dots,$ and $x_g(k)$ is F_{ig}, THEN

$$\begin{aligned}\mathbf{x}(k+1) &= A_i\mathbf{x}(k) + \sum_{j=1}^{q} A_{ji}^d \mathbf{x}(k-d_j) + B\mathbf{w}(k) \\ \mathbf{y}(k) &= C_i\mathbf{x}(k) + \sum_{j=1}^{q} C_{ji}^d \mathbf{x}(k-d_j) + G\mathbf{v}(k) \\ \mathbf{s}(k) &= L\mathbf{x}(k) + \sum_{j=1}^{q} L_{dj}\mathbf{x}(k-d_j)\end{aligned} \tag{2}$$

where $i = 1, 2, \dots, N$; N is the number of fuzzy rules used in the T–S fuzzy model; g is the number of premise variables; F_{ij} is a fuzzy subset; $A_i, A_{ji}^d \in R^{n\times n}; C_i, C_{ji}^d \in R^{m\times n}; k = 0, \dots, k_f - 1$; and k_f is the final discrete time. The fuzzy system (2) is supposed to have singleton fuzzifier, product inference, and centroid defuzzifier.

The nonlinear system (1) can be described as

$$\begin{aligned}\mathbf{x}(k+1) &= F(\mathbf{x}(k)) + F_d(\mathbf{x}(k, \mathbf{d}, q)) + B\mathbf{w}(k) \\ &= \sum_{i=1}^{N} h_i(\mathbf{x}(k)) \left[A_i\mathbf{x}(k) + \sum_{j=1}^{q} A_{ji}^d\mathbf{x}(k-d_j)\right] \\ &\quad + B\mathbf{w}(k) + F(\mathbf{x}(k)) + F_d(\mathbf{x}(k, \mathbf{d}, q)) \\ &\quad - \sum_{i=1}^{N} h_i(\mathbf{x}(k)) \left[A_i\mathbf{x}(k) + \sum_{j=1}^{q} A_{ji}^d\mathbf{x}(k-d_j)\right] \\ &= \sum_{i=1}^{N} h_i(\mathbf{x}(k)) \left[A_i\mathbf{x}(k) + \sum_{j=1}^{q} A_{ji}^d\mathbf{x}(k-d_j)\right] \\ &\quad + B\mathbf{w}(k) + \Delta F(\mathbf{x}(k)) + \Delta F_d(\mathbf{x}(k, \mathbf{d}, q))\end{aligned} \tag{3}$$

where for $i = 1, 2, \dots, N$

$$\begin{aligned}h_i(\mathbf{x}(k)) &= \frac{\mu_i(\mathbf{x}(k))}{\sum_{i=1}^{N} \mu_i(\mathbf{x}(k))} \\ \mu_i(\mathbf{x}(k)) &= \prod_{j=1}^{g} F_{ij}(x_j(k))\end{aligned} \tag{4}$$

and $F_{ij}(x_j(k))$ is the membership function of $x_j(k)$ in F_{ij}. Here $\mu_i(\mathbf{x}(k)) \ge 0$. In (3)

$$\Delta F(\mathbf{x}(k)) = F(\mathbf{x}(k)) - \sum_{i=1}^{N} h_i(\mathbf{x}(k))A_i\mathbf{x}(k) \tag{5}$$

and

$$\begin{aligned}\Delta F_d(\mathbf{x}(k, \mathbf{d}, q)) &= F_d(\mathbf{x}(k, \mathbf{d}, q)) \\ &\quad - \sum_{i=1}^{N} h_i(\mathbf{x}(k)) \sum_{j=1}^{q} A_{ji}^d\mathbf{x}(k-d_j).\end{aligned} \tag{6}$$

We assume that

$$\sum_{i=1}^{N} \mu_i(\mathbf{x}(k)) > 0.$$

Therefore

$$\sum_{i=1}^{N} h_i(\mathbf{x}(k)) = 1.$$

The output of the nonlinear system (1) can be described as

$$\begin{aligned}\mathbf{y}(k) &= H(\mathbf{x}(k)) + H_d(\mathbf{x}(k,\mathbf{d},q)) + G\mathbf{v}(k)\\ &= \sum_{i=1}^{N} h_i(\mathbf{x}(k)) \left[C_i\mathbf{x}(k) + \sum_{j=1}^{q} C_{ji}^d \mathbf{x}(k-d_j)\right]\\ &\quad + G\mathbf{v}(k) + H(\mathbf{x}(k)) + H_d(\mathbf{x}(k,\mathbf{d},q))\\ &\quad - \sum_{i=1}^{N} h_i(\mathbf{x}(k)) \left[C_i\mathbf{x}(k) + \sum_{j=1}^{q} C_{ji}^d \mathbf{x}(k-d_j)\right]\\ &= \sum_{i=1}^{N} h_i(\mathbf{x}(k)) \left[C_i\mathbf{x}(k) + \sum_{j=1}^{q} C_{ji}^d \mathbf{x}(k-d_j)\right]\\ &\quad + G\mathbf{v}(k) + \Delta H(\mathbf{x}(k)) + \Delta H_d(\mathbf{x}(k,\mathbf{d},q))\end{aligned}$$

where

$$\Delta H(\mathbf{x}(k)) = H(\mathbf{x}(k)) - \sum_{i=1}^{N} h_i(\mathbf{x}(k)) C_i \mathbf{x}(k) \tag{7}$$

$$\begin{aligned}\Delta H_d(\mathbf{x}(k,\mathbf{d},q)) &= H_d(\mathbf{x}(k,\mathbf{d},q))\\ &\quad - \sum_{i=1}^{N} h_i(\mathbf{x}(k)) \sum_{j=1}^{q} C_{ji}^d \mathbf{x}(k-d_j).\end{aligned} \tag{8}$$

Here, $\Delta F(\mathbf{x}(k))$, $\Delta F_d(\mathbf{x}(k,\mathbf{d},q))$, $\Delta H(\mathbf{x}(k))$, and $\Delta H_d(\mathbf{x}(k,\mathbf{d},q))$ denote the approximation errors between the system (1) and the T–S fuzzy model (2). We assume that there exist Ω, Ω_d, Ψ, and Ψ_d such that the following inequalities hold for all $\mathbf{x}(k)$:

$$\begin{aligned}&\Delta F^T(\mathbf{x}(k))\Delta F(\mathbf{x}(k))\\ &\quad \le \mathbf{x}^T(k)\Omega\mathbf{x}(k)\end{aligned} \tag{9}$$

$$\begin{aligned}&\Delta F_d^T(\mathbf{x}(k,\mathbf{d},q))\Delta F_d(\mathbf{x}(k,\mathbf{d},q))\\ &\quad \le \sum_{j=1}^{q} \mathbf{x}^T(k-d_j)\Omega_{dj}\mathbf{x}(k-d_j)\end{aligned} \tag{10}$$

$$\begin{aligned}&\Delta H^T(\mathbf{x}(k))\Delta H(\mathbf{x}(k))\\ &\quad \le \mathbf{x}^T(k)\Psi\mathbf{x}(k)\end{aligned} \tag{11}$$

$$\begin{aligned}&\Delta H_d^T(\mathbf{x}(k,\mathbf{d},q))\Delta H_d(\mathbf{x}(k,\mathbf{d},q))\\ &\quad \le \sum_{j=1}^{q} \mathbf{x}^T(k-d_j)\Psi_{dj}\mathbf{x}(k-d_j)\end{aligned} \tag{12}$$

where

$$\Omega, \Omega_d = \begin{bmatrix} \Omega_{d1} & 0 & \cdots & 0\\ 0 & \Omega_{d2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \Omega_{dq}\end{bmatrix}, \Psi$$

$$\Psi_d = \begin{bmatrix} \Psi_{d1} & 0 & \cdots & 0\\ 0 & \Psi_{d2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \Psi_{dq}\end{bmatrix}$$

are positive definite symmetric matrices. Inequalities (9)–(12) indicate that the model approximation errors are bounded. Together, the model approximation errors satisfy

$$\begin{aligned}&\Delta F^T(\mathbf{x}(k))\Delta F(\mathbf{x}(k)) + \Delta F_d^T(\mathbf{x}(k,\mathbf{d},q))\Delta F_d(\mathbf{x}(k,\mathbf{d},q))\\ &\quad + \Delta H^T(\mathbf{x}(k))\Delta H(\mathbf{x}(k))\\ &\quad + \Delta H_d^T(\mathbf{x}(k,\mathbf{d},q))\Delta H_d(\mathbf{x}(k,\mathbf{d},q))\\ &\qquad \le \mathbf{x}^T(k)(\Omega+\Psi)\mathbf{x}(k)\\ &\qquad\quad + \sum_{j=1}^{q} \mathbf{x}^T(k-d_j)(\Omega_{dj}+\Psi_{dj})\mathbf{x}(k-d_j).\end{aligned} \tag{13}$$

The T–S fuzzy model is a piecewise interpolation of several linear models at different operating points through fuzzy membership functions [20], [24]. It can be used as an approximation model for a nonlinear system with time delays. We will design a fuzzy H_∞ filter for system (1) to tolerate the approximation model errors which are upper bounded as in (13).

III. Fuzzy H_∞ Filtering Analysis Based on the T–S Fuzzy Model

Based on the fuzzy model (2), a fuzzy H_∞ filter dealing with the signal estimation for the nonlinear system (1) is described as follows.

The lth rule of a fuzzy H_∞ filter based on the T–S fuzzy model is given in the following form.

Rule l: IF $\hat{x}_1(k)$ is $F_{l1}, \ldots$, and $\hat{x}_g(k)$ is F_{lg}, THEN

$$\begin{aligned}\hat{\mathbf{x}}(k+1) &= A_l\hat{\mathbf{x}}(k) + \sum_{j=1}^{q} A_{jl}^d \hat{\mathbf{x}}(k-d_j)\\ &\quad + K_l[\mathbf{y}(k) - \hat{\mathbf{y}}(k)]\\ \hat{\mathbf{s}}(k) &= L\hat{\mathbf{x}}(k) + \sum_{j=1}^{q} L_{dj}\hat{\mathbf{x}}(k-d_j)\end{aligned} \tag{14}$$

where K_l is the fuzzy estimator gain for the lth estimation rule, $l = 1, 2, \ldots, N$; g is the number of premise variables in the T–S model; $\hat{\mathbf{x}}(k) = 0$ for $k = -d_{\max}, -d_{\max}+1, \ldots, -1$, $\hat{\mathbf{x}}(0) = \hat{\mathbf{x}}_0$ are the initial estimates; and

$$\hat{\mathbf{y}}(k) = \sum_{m=1}^{N} h_m(\hat{\mathbf{x}}(k)) \left[C_m\hat{\mathbf{x}}(k) + \sum_{j=1}^{q} C_{jm}^d \hat{\mathbf{x}}(k-d_j)\right].$$

The fuzzy system (14) is supposed to have singleton fuzzifier, product inference, and centroid defuzzifier. The overall fuzzy estimator (14) can be written as

$$\begin{aligned}&\hat{\mathbf{x}}(k+1)\\ &\quad = \sum_{l=1}^{N} h_l(\hat{\mathbf{x}}(k)) \left[A_l\hat{\mathbf{x}}(k) + \sum_{j=1}^{q} A_{jl}^d \hat{\mathbf{x}}(k-d_j)\right.\\ &\qquad \left. + K_l(\mathbf{y}(k) - \hat{\mathbf{y}}(k))\right]\\ &\quad = \sum_{l=1}^{N} h_l(\hat{\mathbf{x}}(k)) \left[A_l\hat{\mathbf{x}}(k) + \sum_{j=1}^{q} A_{jl}^d \hat{\mathbf{x}}(k-d_j)\right.\end{aligned}$$

$$
\begin{aligned}
&+ K_l \left(\sum_{i=1}^{N} h_i(\mathbf{x}(k)) \left(C_i \mathbf{x}(k) + \sum_{j=1}^{q} C_{ji}^d \mathbf{x}(k-d_j) \right) \right. \\
&+ G\mathbf{v}(k) + \Delta H(\mathbf{x}(k)) + \Delta H_d(\mathbf{x}(k,\mathbf{d},q))) \\
&- K_l \sum_{m=1}^{N} h_m(\hat{\mathbf{x}}(k)) \\
&\left. \times \left(C_m \hat{\mathbf{x}}(k) + \sum_{j=1}^{q} C_{jm}^d \hat{\mathbf{x}}(k-d_j) \right) \right] \\
= &\sum_{l=1}^{N} h_l(\hat{\mathbf{x}}(k)) \sum_{i=1}^{N} h_i(\mathbf{x}(k)) \sum_{m=1}^{N} h_m(\hat{\mathbf{x}}(k)) \\
&\times \left[A_l \hat{\mathbf{x}}(k) + \sum_{j=1}^{q} A_{jl}^d \hat{\mathbf{x}}(k-d_j) + K_l C_i \mathbf{x}(k) \right. \\
&+ K_l \sum_{j=1}^{q} C_{ji}^d \mathbf{x}(k-d_j) + K_l G \mathbf{v}(k) \\
&+ K_l \Delta H(\mathbf{x}(k)) + K_l \Delta H_d(\mathbf{x}(k,\mathbf{d},q)) \\
&\left. - K_l C_m \hat{\mathbf{x}}(k) - K_l \sum_{j=1}^{q} C_{jm}^d \hat{\mathbf{x}}(k-d_j) \right]. \qquad (15)
\end{aligned}
$$

From (3) and (15), the augmented system can be written in the following form:

$$
\begin{aligned}
\boldsymbol{\eta}(k+1) = &\sum_{i=1}^{N} h_i(\mathbf{x}(k)) \sum_{l=1}^{N} h_l(\hat{\mathbf{x}}(k)) \sum_{m=1}^{N} h_m(\hat{\mathbf{x}}(k)) \\
&\times \left[\tilde{A}_{ilm} \boldsymbol{\eta}(k) + \sum_{j=1}^{q} \tilde{A}_{jilm}^d \boldsymbol{\eta}(k-d_j) + \tilde{E}_l \Delta\Gamma(k) \right. \\
&\left. + \tilde{E}_l^d \Delta\Gamma_d(k,\mathbf{d},q) + \tilde{G}_l \varpi(k) \right] \qquad (16)
\end{aligned}
$$

where

$$
\boldsymbol{\eta}(k) = \begin{bmatrix} \mathbf{x}(k) \\ \hat{\mathbf{x}}(k) \end{bmatrix} \quad \boldsymbol{\eta}(k-d_j) = \begin{bmatrix} \mathbf{x}(k-d_j) \\ \hat{\mathbf{x}}(k-d_j) \end{bmatrix}
$$

$$
\tilde{A}_{ilm} = \begin{bmatrix} A_i & 0 \\ K_l C_i & A_l - K_l C_m \end{bmatrix}
$$

$$
\tilde{A}_{jilm}^d = \begin{bmatrix} A_{ji}^d & 0 \\ K_l C_{ji}^d & A_{jl}^d - K_l C_{jm}^d \end{bmatrix}
$$

$$
\tilde{E}_l = \begin{bmatrix} I & 0 \\ 0 & K_l \end{bmatrix}
$$

$$
\tilde{E}_l^d = \begin{bmatrix} I & 0 \\ 0 & K_l \end{bmatrix}, \quad \Delta\Gamma(k) = \begin{bmatrix} \Delta F(\mathbf{x}(k)) \\ \Delta H(\mathbf{x}(k)) \end{bmatrix}
$$

$$
\Delta\Gamma_d(k,\mathbf{d},q) = \begin{bmatrix} \Delta F_d(\mathbf{x}(k,\mathbf{d},q)) \\ \Delta H_d(\mathbf{x}(k,\mathbf{d},q)) \end{bmatrix}
$$

$$
\tilde{G}_l = \begin{bmatrix} B & 0 \\ 0 & K_l G \end{bmatrix}, \quad \varpi(k) = \begin{bmatrix} \mathbf{w}(k) \\ \mathbf{v}(k) \end{bmatrix}.
$$

The fuzzy H_∞ filter design problem addressed in this paper is stated as follows.

First, define the estimation error

$$
\mathbf{e}(k) = \mathbf{s}(k) - \hat{\mathbf{s}}(k) = \tilde{L}\boldsymbol{\eta}(k) + \sum_{j=1}^{q} \tilde{L}_{dj} \boldsymbol{\eta}(k-d_j) \qquad (17)
$$

where $\tilde{L} = [L, -L]$, $\tilde{L}_{dj} = [L_{dj}, -L_{dj}]$. From (16) and (17), the filtering error dynamics is described by

$$
\begin{cases}
\boldsymbol{\eta}(k+1) = \tilde{A}\boldsymbol{\eta}(k) + \sum_{j=1}^{q} \tilde{A}_{dj}\boldsymbol{\eta}(k-d_j) \\
\qquad + \tilde{E}\Delta\Gamma(k) + \tilde{E}_d \Delta\Gamma_d(k,\mathbf{d},q) + \tilde{G}\varpi(k) \\
\mathbf{e}(k) = \tilde{L}\boldsymbol{\eta}(k) + \sum_{j=1}^{q} \tilde{L}_{dj}\boldsymbol{\eta}(k-d_j)
\end{cases} \qquad (18)
$$

where

$$
\tilde{A} = \sum_{i=1}^{N} h_i(\mathbf{x}(k)) \sum_{l=1}^{N} h_l(\hat{\mathbf{x}}(k)) \sum_{m=1}^{N} h_m(\hat{\mathbf{x}}(k)) \tilde{A}_{ilm}
$$

$$
\tilde{A}_{dj} = \sum_{i=1}^{N} h_i(\mathbf{x}(k)) \sum_{l=1}^{N} h_l(\hat{\mathbf{x}}(k)) \sum_{m=1}^{N} h_m(\hat{\mathbf{x}}(k)) \tilde{A}_{jilm}^d
$$

$$
\tilde{E} = \sum_{i=1}^{N} h_i(\mathbf{x}(k)) \sum_{l=1}^{N} h_l(\hat{\mathbf{x}}(k)) \sum_{m=1}^{N} h_m(\hat{\mathbf{x}}(k)) \tilde{E}_l
$$

$$
\tilde{E}_d = \sum_{i=1}^{N} h_i(\mathbf{x}(k)) \sum_{l=1}^{N} h_l(\hat{\mathbf{x}}(k)) \sum_{m=1}^{N} h_m(\hat{\mathbf{x}}(k)) \tilde{E}_l^d
$$

$$
\tilde{G} = \sum_{i=1}^{N} h_i(\mathbf{x}(k)) \sum_{l=1}^{N} h_l(\hat{\mathbf{x}}(k)) \sum_{m=1}^{N} h_m(\hat{\mathbf{x}}(k)) \tilde{G}_l.
$$

Next, we define the H_∞ index for the H_∞ filter. Given a scalar $\gamma > 0$ and initial estimation error, the H_∞ index of the H_∞ filter is defined as follows [24]:

$$
\begin{aligned}
&\sum_{k=1}^{k_f} \mathbf{e}^T(k) Q \mathbf{e}(k) \\
&\quad \le \gamma^2 \left[\mathbf{e}^T(0) P_0 \mathbf{e}(0) + \sum_{k=0}^{k_f-1} (\mathbf{w}^T(k) W_w \mathbf{w}(k) \right. \\
&\qquad \left. + \mathbf{v}(k)^T W_v \mathbf{v}(k)) \right] \qquad (19)
\end{aligned}
$$

where $Q \in R^{p\times p}$, $P_0 \in R^{p\times p}$, $W_w \in R^{n\times n}$, and $W_v \in R^{m\times m}$ are positive definite weighting matrices.

Remark 3.1: The fuzzy H_∞ filter design problem is to design an asymptotically stable nonlinear filter of the form of (14) such that the filtering error system (18) is globally asymptotically stable and satisfies (19). Note that (19) denotes that $\mathbf{e}(k)$ must be less than a prescribed level γ^2 for all possible $\mathbf{e}(0)$, $\mathbf{w}(k)$ and $\mathbf{v}(k)$ from the energy point of view [24]. ■

Inequality (19) can be rewritten in the following form:

$$
\begin{aligned}
&\sum_{k=1}^{k_f} \mathbf{e}^T(k) Q \mathbf{e}(k) \\
&\quad = \sum_{k=1}^{k_f} \left[\tilde{L}\boldsymbol{\eta}(k) + \sum_{j=1}^{q} \tilde{L}_{dj}\boldsymbol{\eta}(k-d_j) \right]^T \\
&\qquad \times Q \left[\tilde{L}\boldsymbol{\eta}(k) + \sum_{j=1}^{q} \tilde{L}_{dj}\boldsymbol{\eta}(k-d_j) \right] \\
&\quad \le \gamma^2 \left[\mathbf{e}^T(0) P_0 \mathbf{e}(0) + \sum_{k=0}^{k_f-1} (\mathbf{w}^T(k) W_w \mathbf{w}(k) \right. \\
&\qquad \left. + \mathbf{v}(k)^T W_v \mathbf{v}(k)) \right]. \qquad (20)
\end{aligned}
$$

Define

$$\boldsymbol{\xi}(k,\mathbf{d}) = [\boldsymbol{\eta}^T(k), \quad \boldsymbol{\eta}^T(k-d_1),\ldots,\boldsymbol{\eta}^T(k-d_q)]^T$$

(20) becomes

$$\begin{aligned}
&\sum_{k=1}^{k_f} \mathbf{e}^T(k)Q\mathbf{e}(k)\\
&= \sum_{k=1}^{k_f}\Bigg[\boldsymbol{\eta}^T(k)\tilde{L}^TQ\tilde{L}\boldsymbol{\eta}(k)\\
&\quad + 2\boldsymbol{\eta}^T(k)\tilde{L}^TQ\sum_{j=1}^{q}\tilde{L}_{dj}\boldsymbol{\eta}(k-d_j)\\
&\quad + \sum_{j=1}^{q}\sum_{t=1}^{q}\boldsymbol{\eta}^T(k-d_j)\tilde{L}_{dj}^TQ\tilde{L}_{dt}\boldsymbol{\eta}(k-d_t)\Bigg]\\
&= \sum_{k=1}^{k_f}\boldsymbol{\xi}^T(k,\mathbf{d})\tilde{Q}\boldsymbol{\xi}(k,\mathbf{d})\\
&\le \gamma^2\Bigg[\mathbf{e}^T(0)P_0\mathbf{e}(0) + \sum_{k=0}^{k_f-1}(\mathbf{w}^T(k)W_w\mathbf{w}(k)\\
&\qquad + \mathbf{v}(k)^TW_v\mathbf{v}(k))\Bigg]\\
&= \gamma^2\Bigg[\boldsymbol{\eta}^T(0)\tilde{L}^TP_0\tilde{L}\boldsymbol{\eta}(0) + \sum_{k=0}^{k_f-1}(\mathbf{w}^T(k)W_w\mathbf{w}(k)\\
&\qquad + \mathbf{v}(k)^TW_v\mathbf{v}(k))\Bigg]\\
&= \gamma^2\Bigg[\boldsymbol{\xi}^T(0,\mathbf{d})\Lambda^T\tilde{L}^TP_0\tilde{L}\Lambda\boldsymbol{\xi}(0,\mathbf{d})\\
&\qquad + \sum_{k=0}^{k_f-1}(\mathbf{w}^T(k)W_w\mathbf{w}(k) + \mathbf{v}(k)^TW_v\mathbf{v}(k))\Bigg]\\
&= \gamma^2\Bigg[\boldsymbol{\xi}^T(0,\mathbf{d})\tilde{P}_0\boldsymbol{\xi}(0,\mathbf{d}) + \sum_{k=0}^{k_f-1}(\mathbf{w}^T(k)W_w\mathbf{w}(k)\\
&\qquad + \mathbf{v}(k)^TW_v\mathbf{v}(k))\Bigg] \qquad (21)
\end{aligned}$$

where

$$\begin{aligned}
\tilde{Q} &= \begin{bmatrix}\tilde{Q}_{11} & \tilde{Q}_{12}\\ \tilde{Q}_{21} & \tilde{Q}_{22}\end{bmatrix}, \quad \tilde{Q}_{11} = \tilde{L}^TQ\tilde{L}\\
\tilde{Q}_{12} &= [\tilde{L}^TQ\tilde{L}_{d1}, \quad \tilde{L}^TQ\tilde{L}_{d2},\ldots,\tilde{L}^TQ\tilde{L}_{dq}]\\
\tilde{Q}_{21} &= \tilde{Q}_{12}^T\\
\tilde{Q}_{22} &= \begin{bmatrix}\tilde{L}_{d1}^TQ\tilde{L}_{d1} & \tilde{L}_{d1}^TQ\tilde{L}_{d2} & \cdots & \tilde{L}_{d1}^TQ\tilde{L}_{dq}\\ * & \tilde{L}_{d2}^TQ\tilde{L}_{d2} & \cdots & \tilde{L}_{d2}^TQ\tilde{L}_{dq}\\ \vdots & \vdots & \ddots & \vdots\\ * & * & \cdots & \tilde{L}_{dq}^TQ\tilde{L}_{dq}\end{bmatrix}
\end{aligned}$$

$\Lambda = [I_{2n\times 2n} \;\; 0_{2n\times 2nq}]$ (thus, $\eta(k) = \Lambda\boldsymbol{\xi}(k,\mathbf{d})$), $\tilde{P}_0 = \Lambda^T\tilde{L}^TP_0\tilde{L}\Lambda$, and $*$ represents the corresponding symmetric element of matrix $\tilde{Q}_{22}$. Therefore, the H_∞ index defined in (19) can be modified as in (21).

The next lemma is useful in establishing our main result, Theorem 3.1.

Lemma 3.1 (Schur Complements for Strict Inequalities [1]): The LMI

$$\begin{bmatrix}Q(x) & S(x)\\ S^T(x) & R(x)\end{bmatrix} > 0$$

where $Q(x) = Q^T(x)$, $R(x) = R^T(x)$, and $S(x)$ depends affinely on x, is equivalent to

$$R(x) > 0, \quad Q(x) - S(x)R^{\dagger}(x)S^T(x) > 0$$

where $R^{\dagger}(x)$ denotes the Moore–Penrose inverse of matrix $R(x)$. ■

The following theorem establishes the existence of fuzzy H_∞ filter and guarantees that the filtering error system satisfies the H_∞ index and is asymptotically stable.

Theorem 3.1: For nonlinear system (1) and a prescribed real number $\gamma > 0$, if there exist a positive scalar $\tau > 0$ and symmetric positive definite matrices $P \in R^{2n\times 2n}$ and $P_j \in R^{2n\times 2n}$ such that the matrix inequalities shown in (22) at the bottom of the page hold for $h_i(\mathbf{x}(k))\cdot h_l(\hat{\mathbf{x}}(k))\cdot h_m(\hat{\mathbf{x}}(k)) \neq 0$, $i,l,m = 1,2,\ldots,N$, where

$$\begin{aligned}
\tilde{A}_d &= [\tilde{A}_{d1},\tilde{A}_{d2},\ldots,\tilde{A}_{dq}]\\
\tilde{L}_d &= [\tilde{L}_{d1},\tilde{L}_{d2},\ldots,\tilde{L}_{dq}]\\
S &= \begin{bmatrix}P_1 & 0 & \cdots & 0\\ 0 & P_2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & P_q\end{bmatrix}, \quad \Theta = \begin{bmatrix}\Omega+\Psi & 0\\ 0 & 0\end{bmatrix}
\end{aligned}$$

$$\begin{bmatrix}
-P+\sum_{j=1}^{q}P_j+\tau\Theta & 0 & 0 & 0 & 0 & \tilde{A}^TP & \tilde{L}^T\\
0 & -S+\tau\Theta_d & 0 & 0 & 0 & \tilde{A}_d^TP & \tilde{L}_d^T\\
0 & 0 & -\tau I & 0 & 0 & \tilde{E}^TP & 0\\
0 & 0 & 0 & -\tau I & 0 & \tilde{E}_d^TP & 0\\
0 & 0 & 0 & 0 & -\gamma^2W & \tilde{G}^TP & 0\\
P\tilde{A} & P\tilde{A}_d & P\tilde{E} & P\tilde{E}_d & P\tilde{G} & -P & 0\\
\tilde{L} & \tilde{L}_d & 0 & 0 & 0 & 0 & -Q^{-1}
\end{bmatrix} < 0 \qquad (22)$$

$$\Theta_d = \begin{bmatrix} \Theta_{d1} & 0 & \cdots & 0 \\ 0 & \Theta_{d2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Theta_{dq} \end{bmatrix}$$
$$\Theta_{dj} = \begin{bmatrix} \Omega_{dj} + \Psi_{dj} & 0 \\ 0 & 0 \end{bmatrix}, \quad W = \begin{bmatrix} W_w & 0 \\ 0 & W_v \end{bmatrix}$$

and Q is defined as in (21), then the filtering error system (18) is asymptotically stable and satisfies the H_∞ index (21).

Proof: We first show that (22) guarantees that the filtering error system (18) is globally asymptotically stable. We choose the following Lyapunov function candidate similarly as in [4] and [5]

$$V(\boldsymbol{\eta}(k)) = \boldsymbol{\eta}^T(k) P \boldsymbol{\eta}(k) + \sum_{j=1}^{q} \sum_{l=k-d_j}^{k-1} \boldsymbol{\eta}^T(l) P_j \boldsymbol{\eta}(l) \qquad (23)$$

where $P \in R^{2n\times 2n}$ and $P_j \in R^{2n\times 2n}$ are positive definite symmetric matrices.

Along the trajectories of system (18) with $\varpi(k) = 0$, the corresponding forward difference is calculated as

$$\begin{aligned}
\Delta V(k) &= V(\boldsymbol{\eta}(k+1)) - V(\boldsymbol{\eta}(k)) \\
&= \boldsymbol{\eta}^T(k+1) P \boldsymbol{\eta}(k+1) \\
&\quad + \sum_{j=1}^{q} \sum_{l=k-d_j+1}^{k} \boldsymbol{\eta}^T(l) P_j \boldsymbol{\eta}(l) \\
&\quad - \sum_{j=1}^{q} \sum_{l=k-d_j}^{k-1} \boldsymbol{\eta}^T(l) P_j \boldsymbol{\eta}(l) - \boldsymbol{\eta}^T(k) P \boldsymbol{\eta}(k) \\
&= \left[\tilde{A}\boldsymbol{\eta}(k) + \sum_{j=1}^{q} \tilde{A}_{dj} \boldsymbol{\eta}(k-d_j) + \tilde{E}\Delta\Gamma(k) \right. \\
&\qquad \left. + \tilde{E}_d \Delta\Gamma_d(k, \mathbf{d}, q) \right]^T P \\
&\quad \times \left[\tilde{A}\boldsymbol{\eta}(k) + \sum_{j=1}^{q} \tilde{A}_{dj} \boldsymbol{\eta}(k-d_j) \right. \\
&\qquad \left. + \tilde{E}\Delta\Gamma(k) + \tilde{E}_d \Delta\Gamma_d(k, \mathbf{d}, q) \right] \\
&\quad + \sum_{j=1}^{q} [\boldsymbol{\eta}^T(k) P_j \boldsymbol{\eta}(k) - \boldsymbol{\eta}^T(k-d_j) P_j \boldsymbol{\eta}(k-d_j)] \\
&\quad - \boldsymbol{\eta}^T(k) P \boldsymbol{\eta}(k) + \tau \Delta\Gamma^T(k) \Delta\Gamma(k) \\
&\quad + \tau \Delta\Gamma_d^T(k, \mathbf{d}, q) \Delta\Gamma_d(k, \mathbf{d}, q) \\
&\quad - \tau \Delta\Gamma^T(k) \Delta\Gamma(k) \\
&\quad - \tau \Delta\Gamma_d^T(k, \mathbf{d}, q) \Delta\Gamma_d(k, \mathbf{d}, q). \qquad (24)
\end{aligned}$$

By inequality (13), (24) becomes

$$\begin{aligned}
\Delta V &\le \left[\tilde{A}\boldsymbol{\eta}(k) + \sum_{j=1}^{q} \tilde{A}_{dj} \boldsymbol{\eta}(k-d_j) + \tilde{E}\Delta\Gamma(k) \right. \\
&\qquad \left. + \tilde{E}_d \Delta\Gamma_d(k, \mathbf{d}, q) \right]^T \\
&\quad P \left[\tilde{A}\boldsymbol{\eta}(k) \right. \\
&\qquad + \sum_{j=1}^{q} \tilde{A}_{dj} \boldsymbol{\eta}(k-d_j) \\
&\qquad \left. + \tilde{E}\Delta\Gamma(k) + \tilde{E}_d \Delta\Gamma_d(k, \mathbf{d}, q) \right] \\
&\quad + \sum_{j=1}^{q} [\boldsymbol{\eta}^T(k) P_j \boldsymbol{\eta}(k) \\
&\quad - \boldsymbol{\eta}^T(k-d_j) P_j \boldsymbol{\eta}(k-d_j)] - \boldsymbol{\eta}^T(k) P \boldsymbol{\eta}(k) \\
&\quad + \tau \mathbf{x}^T(k)(\Omega + \Psi)\mathbf{x}(k) \\
&\quad + \tau \sum_{j=1}^{q} \mathbf{x}^T(k-d_j)(\Omega_{dj} + \Psi_{dj})\mathbf{x}(k-d_j) \\
&\quad - \tau \Delta\Gamma^T(k)\Delta\Gamma(k) - \tau \Delta\Gamma_d^T(k, \mathbf{d}, q)\Delta\Gamma_d(k, \mathbf{d}, q) \\
&= \left[\tilde{A}\boldsymbol{\eta}(k) + \sum_{j=1}^{q} \tilde{A}_{dj} \boldsymbol{\eta}(k-d_j) + \tilde{E}\Delta\Gamma(k) \right. \\
&\qquad \left. + \tilde{E}_d \Delta\Gamma_d(k, \mathbf{d}, q) \right]^T \\
&\qquad \times P \left[\tilde{A}\boldsymbol{\eta}(k) + \sum_{j=1}^{q} \tilde{A}_{dj} \boldsymbol{\eta}(k-d_j) \right. \\
&\qquad\qquad \left. + \tilde{E}\Delta\Gamma(k) + \tilde{E}_d \Delta\Gamma_d(k, \mathbf{d}, q) \right] \\
&\qquad + \sum_{j=1}^{q} [\boldsymbol{\eta}^T(k) P_j \boldsymbol{\eta}(k) - \boldsymbol{\eta}^T(k-d_j) P_j \boldsymbol{\eta}(k-d_j)] \\
&\quad - \boldsymbol{\eta}^T(k) P \boldsymbol{\eta}(k) + \tau \boldsymbol{\eta}^T(k) \Theta \boldsymbol{\eta}(k) \\
&\quad + \tau \sum_{j=1}^{q} \boldsymbol{\eta}^T(k-d_j) \Theta_{dj} \boldsymbol{\eta}(k-d_j) - \tau \Delta\Gamma^T(k)\Delta\Gamma(k) \\
&\quad - \tau \Delta\Gamma_d^T(k, \mathbf{d}, q)\Delta\Gamma_d(k, \mathbf{d}, q) \\
&= -\boldsymbol{\zeta}^T(k, \mathbf{d}) M \boldsymbol{\zeta}(k, \mathbf{d}) \qquad (25)
\end{aligned}$$

where $\tau > 0$

$$\boldsymbol{\zeta}(k, \mathbf{d}) = [\boldsymbol{\eta}^T(k), \boldsymbol{\eta}^T(k, \mathbf{d}, q), \Delta\Gamma^T(k), \Delta\Gamma_d^T(k, \mathbf{d}, q)]^T,$$
$$\boldsymbol{\eta}^T(k, \mathbf{d}, q) \triangleq [\boldsymbol{\eta}^T(k-d_1), \boldsymbol{\eta}^T(k-d_2), \ldots, \boldsymbol{\eta}^T(k-d_q)],$$
$$M = -\begin{bmatrix} \tilde{A}^T \\ \tilde{A}_d^T \\ \tilde{E}^T \\ \tilde{E}_d^T \end{bmatrix} P [\tilde{A} \quad \tilde{A}_d \quad \tilde{E} \quad \tilde{E}_d]$$

$$-\begin{bmatrix} -P+\sum_{j=1}^{q}P_j & 0 & 0 & 0\\ 0 & -S & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix} - \begin{bmatrix} \tau\Theta & 0 & 0 & 0\\ 0 & \tau\Theta_d & 0 & 0\\ 0 & 0 & -\tau I & 0\\ 0 & 0 & 0 & -\tau I\end{bmatrix}$$

and $\tilde{A}_d^T, \tilde{E}_d^T, S, \Theta, \Theta_d, \Theta_{dj}$ are defined as in (22).

The sufficient condition for

$$\Delta V(k) \le -\boldsymbol{\zeta}^T(k,\mathbf{d})M\boldsymbol{\zeta}(k,\mathbf{d}) < 0$$

is as follows:

$$\begin{bmatrix}\tilde{A}^T\\ \tilde{A}_d^T\\ \tilde{E}^T\\ \tilde{E}_d^T\end{bmatrix} P[\tilde{A}\quad \tilde{A}_d\quad \tilde{E}\quad \tilde{E}_d] + \begin{bmatrix} -P+\sum_{j=1}^{q}P_j+\tau\Theta & 0 & 0 & 0\\ 0 & -S+\tau\Theta_d & 0 & 0\\ 0 & 0 & -\tau I & 0\\ 0 & 0 & 0 & -\tau I\end{bmatrix} < 0. \tag{26}$$

According to the Schur complements in Lemma 3.1, (26) is equivalent to (27) at the bottom of the page. The inequality (22) can easily be verified to guarantee (27). Therefore, it can be concluded that the filtering error system is globally asymptotically stable if (22) holds.

We now show that the filtering error system (18) satisfies the H_∞ index (21).

In order to determine a sufficient condition for the filtering error system (18) to satisfy the H_∞ index (21) under (13), we consider an arbitrary nonzero $\varpi(k) \in l_2[0,+\infty)$ [4] and, following similar steps as in (24) and (25), we derive

$$\begin{aligned}
&\Delta V(k) + \mathbf{e}^T(k)Q\mathbf{e}(k) - \gamma^2(\mathbf{w}^T(k)W_w\mathbf{w}(k) + \mathbf{v}^T(k)W_v\mathbf{v}(k))\\
&\le \left[\tilde{A}\boldsymbol{\eta}(k) + \sum_{j=1}^{q}\tilde{A}_{dj}\boldsymbol{\eta}(k-d_j) + \tilde{E}\Delta\Gamma(k) + \tilde{E}_d\Delta\Gamma_d(k,\mathbf{d},q) + \tilde{G}\varpi(k)\right]^T\\
&\quad\times P\left[\tilde{A}\boldsymbol{\eta}(k) + \sum_{j=1}^{q}\tilde{A}_{dj}\boldsymbol{\eta}(k-d_j) + \tilde{E}\Delta\Gamma(k) + \tilde{E}_d\Delta\Gamma_d(k,\mathbf{d},q) + \tilde{G}\varpi(k)\right]\\
&\quad+\sum_{j=1}^{q}[\boldsymbol{\eta}^T(k)P_j\boldsymbol{\eta}(k) - \boldsymbol{\eta}^T(k-d_j)P_j\boldsymbol{\eta}(k-d_j)] - \boldsymbol{\eta}^T(k)P\boldsymbol{\eta}(k)\\
&\quad+\left[\tilde{L}\boldsymbol{\eta}(k) + \sum_{j=1}^{q}\tilde{L}_{dj}\boldsymbol{\eta}(k-d_j)\right]^T Q\left[\tilde{L}\boldsymbol{\eta}(k)\sum_{j=1}^{q}\tilde{L}_{dj}\boldsymbol{\eta}(k-d_j)\right] - \gamma^2\left[\mathbf{w}^T(k)W_w\mathbf{w}(k) + \mathbf{v}^T(k)W_v\mathbf{v}(k)\right] + \tau\boldsymbol{\eta}^T(k)\Theta\boldsymbol{\eta}(k)\\
&\quad+\tau\sum_{j=1}^{q}\boldsymbol{\eta}^T(k-d_j)\Theta_{dj}\boldsymbol{\eta}(k-d_j) - \tau\Delta\Gamma^T(k)\Delta\Gamma(k) - \tau\Delta\Gamma_d^T(k,\mathbf{d},q)\Delta\Gamma_d(k,\mathbf{d},q)\\
&\le \boldsymbol{\chi}^T(k,\mathbf{d})\Pi\boldsymbol{\chi}(k,\mathbf{d})
\end{aligned}\tag{28}$$

where $\boldsymbol{\chi}(k,\mathbf{d}$ and Π are defined at the bottom of the next page. We have used the notation defined in (16) and (22) when deriving (28). According to Schur complements in Lemma 3.1, $\Pi < 0$ is equivalent to (22).

According to (22), (28) becomes

$$\Delta V(k) + \mathbf{e}^T(k)Q\mathbf{e}(k) - \gamma^2(\mathbf{w}^T(k)W_w\mathbf{w}(k) + \mathbf{v}^T(k)W_v\mathbf{v}(k)) \le 0. \tag{29}$$

From (29), we obtain

$$\sum_{k=0}^{k_f-1}\mathbf{e}^T(k)Q\mathbf{e}(k) + \sum_{k=0}^{k_f-1}\Delta V(k) - \gamma^2\sum_{k=0}^{k_f-1}(\mathbf{w}^T(k)W_w\mathbf{w}(k) + \mathbf{v}^T(k)W_v\mathbf{v}(k)) \le 0$$

which implies that

$$\sum_{k=0}^{k_f-1}\mathbf{e}^T(k)Q\mathbf{e}(k) + \sum_{k=0}^{k_f-1}\Delta V(k) \le \gamma^2\sum_{k=0}^{k_f-1}(\mathbf{w}^T(k)W_w\mathbf{w}(k) + \mathbf{v}^T(k)W_v\mathbf{v}(k)). \tag{30}$$

$$\begin{bmatrix} -P+\sum_{j=1}^{q}P_j+\tau\Theta & 0 & 0 & 0 & \tilde{A}^TP\\ 0 & -S+\tau\Theta_d & 0 & 0 & \tilde{A}_d^TP\\ 0 & 0 & -\tau I & 0 & \tilde{E}^TP\\ 0 & 0 & 0 & -\tau I & \tilde{E}_d^TP\\ P\tilde{A} & P\tilde{A}_d & P\tilde{E} & P\tilde{E}_d & -P\end{bmatrix} < 0. \tag{27}$$

According to the definitions of $\tilde{Q}$ and $\xi(k,\mathbf{d})$, we have

$$\begin{aligned}\sum_{k=1}^{k_f}\mathbf{e}^T(k)Q\mathbf{e}(k) &= \sum_{k=1}^{k_f}\boldsymbol{\xi}^T(k,\mathbf{d})\tilde{Q}\boldsymbol{\xi}(k,\mathbf{d}) \\ &= -\boldsymbol{\xi}^T(0,\mathbf{d})\tilde{Q}\boldsymbol{\xi}(0,\mathbf{d}) \\ &\quad+\boldsymbol{\xi}^T(k_f,\mathbf{d})\tilde{Q}\boldsymbol{\xi}(k_f,\mathbf{d}) \\ &\quad+\sum_{k=0}^{k_f-1}\boldsymbol{\xi}^T(k,\mathbf{d})\tilde{Q}\boldsymbol{\xi}(k,\mathbf{d}) \\ &\quad+\sum_{k=0}^{k_f-1}\Bigg[\boldsymbol{\eta}^T(k+1)P\boldsymbol{\eta}(k+1) \\ &\quad-\boldsymbol{\eta}^T(k)P\boldsymbol{\eta}(k)+\sum_{j=1}^{q}(\boldsymbol{\eta}^T(k)P_j\boldsymbol{\eta}(k) \\ &\quad-\boldsymbol{\eta}^T(k-d_j)P_j\boldsymbol{\eta}(k-d_j))\Bigg] \\ &\quad-\boldsymbol{\eta}^T(k_f)P\boldsymbol{\eta}(k_f)+\boldsymbol{\eta}^T(0)P\boldsymbol{\eta}(0) \\ &\quad-\sum_{k=0}^{k_f-1}\sum_{j=1}^{q}[\boldsymbol{\eta}^T(k)P_j\boldsymbol{\eta}(k) \\ &\quad-\boldsymbol{\eta}^T(k-d_j)P_j\boldsymbol{\eta}(k-d_j)].\end{aligned} \tag{31}$$

By (24), (31) becomes

$$\begin{aligned}\sum_{k=1}^{k_f}\mathbf{e}^T(k)Q\mathbf{e}(k) &= -\boldsymbol{\xi}^T(0,\mathbf{d})\tilde{Q}\boldsymbol{\xi}(0,\mathbf{d}) \\ &\quad+\boldsymbol{\xi}^T(k_f,\mathbf{d})\tilde{Q}\boldsymbol{\xi}(k_f,\mathbf{d}) \\ &\quad+\sum_{k=0}^{k_f-1}\mathbf{e}^T(k)Q\mathbf{e}(k)+\sum_{k=0}^{k_f-1}\Delta V(k) \\ &\quad-\boldsymbol{\eta}^T(k_f)P\boldsymbol{\eta}(k_f)+\boldsymbol{\eta}^T(0)P\boldsymbol{\eta}(0) \\ &\quad-\sum_{k=0}^{k_f}\sum_{j=1}^{q}[\boldsymbol{\eta}^T(k)P_j\boldsymbol{\eta}(k) \\ &\quad-\boldsymbol{\eta}^T(k-d_j)P_j\boldsymbol{\eta}(k-d_j)] \\ &\quad+\sum_{j=1}^{q}(\boldsymbol{\eta}^T(k_f)P_j\boldsymbol{\eta}(k_f) \\ &\quad-\boldsymbol{\eta}^T(k_f-d_j)P_j\boldsymbol{\eta}(k_f-d_j)) \\ &\le \boldsymbol{\xi}^T(k_f,\mathbf{d})\tilde{Q}\boldsymbol{\xi}(k_f,\mathbf{d}) \\ &\quad+\sum_{k=0}^{k_f-1}\mathbf{e}^T(k)Q\mathbf{e}(k)+\sum_{k=0}^{k_f-1}\Delta V(k) \\ &\quad-\boldsymbol{\eta}^T(k_f)P\boldsymbol{\eta}(k_f)+\boldsymbol{\eta}^T(0)P\boldsymbol{\eta}(0) \\ &\quad-\sum_{k=0}^{k_f}\sum_{j=1}^{q}[\boldsymbol{\eta}^T(k)P_j\boldsymbol{\eta}(k) \\ &\quad-\boldsymbol{\eta}^T(k-d_j)P_j\boldsymbol{\eta}(k-d_j)] \\ &\quad+\sum_{j=1}^{q}(\boldsymbol{\eta}^T(k_f)P_j\boldsymbol{\eta}(k_f) \\ &\quad-\boldsymbol{\eta}^T(k_f-d_j)P_j\boldsymbol{\eta}(k_f-d_j)) \\ &= \boldsymbol{\eta}^T(0)P\boldsymbol{\eta}(0)+\boldsymbol{\xi}^T(k_f,\mathbf{d}) \\ &\quad\times\Bigg[\tilde{Q}-\Lambda^TP\Lambda \\ &\quad+\Lambda^T\sum_{j=1}^{q}P_j\Lambda-\Upsilon^TS\Upsilon\Bigg]\boldsymbol{\xi}(k_f,\mathbf{d}) \\ &\quad+\sum_{k=0}^{k_f-1}\mathbf{e}^T(k)Q\mathbf{e}(k)+\sum_{k=0}^{k_f-1}\Delta V(k) \\ &\quad-\sum_{k=0}^{k_f}\sum_{j=1}^{q}[\boldsymbol{\eta}^T(k)P_j\boldsymbol{\eta}(k) \\ &\quad-\boldsymbol{\eta}^T(k-d_j)P_j\boldsymbol{\eta}(k-d_j)]\end{aligned} \tag{32}$$

$$\boldsymbol{\chi}(k,\mathbf{d})=[\boldsymbol{\eta}^T(k)\quad \boldsymbol{\eta}^T(k,\mathbf{d},q)\quad \Delta\Gamma^T(k)\quad \Delta\Gamma_d^T(k,\mathbf{d},q)\quad \varpi^T(k)]^T$$

$$\begin{aligned}\Pi &= \begin{bmatrix}\tilde{A}^T\\ \tilde{A}_d^T\\ \tilde{E}^T\\ \tilde{E}_d^T\\ \tilde{G}^T\end{bmatrix}P[\tilde{A}\quad \tilde{A}_d\quad \tilde{E}\quad \tilde{E}_d\quad \tilde{G}] \\ &\quad+\begin{bmatrix}-P+\sum_{j=1}^{q}P_j+\tau\Theta & 0 & 0 & 0 & 0\\ 0 & -S+\tau\Theta_d & 0 & 0 & 0\\ 0 & 0 & -\tau I & 0 & 0\\ 0 & 0 & 0 & -\tau I & 0\\ 0 & 0 & 0 & 0 & -\gamma^2W\end{bmatrix} \\ &\quad+\begin{bmatrix}\tilde{L}^T\\ \tilde{L}_d^T\\ 0\\ 0\\ 0\end{bmatrix}Q[\tilde{L}\quad \tilde{L}_d\quad 0\quad 0\quad 0]\end{aligned}$$

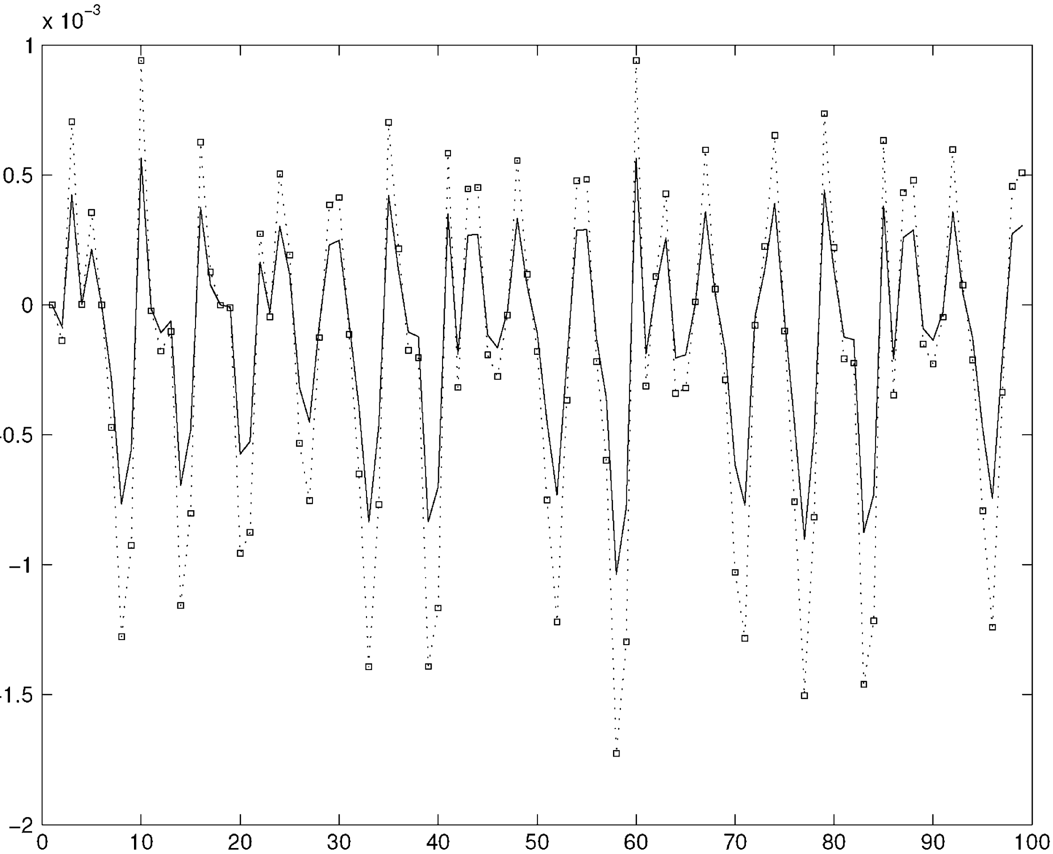

Fig. 1. The estimation error of $x_1(k) - \hat{x}_1(k)$ [the present method (solid line) and EKF (dash-dot line with square marker)].

where $\Lambda = [I_{2n\times 2n}\ \ 0_{2n\times 2nq}]$ and $\Upsilon = [0_{2nq\times 2n}\ \ I_{2nq\times 2nq}]$ (thus, $\boldsymbol{\eta}(k) = \Lambda\boldsymbol{\xi}(k,\mathbf{d})$ and $[\boldsymbol{\eta}^T(k-d_1), \boldsymbol{\eta}^T(k-d_2)\ldots, \boldsymbol{\eta}^T(k-d_q)]^T = \Upsilon\boldsymbol{\xi}(k,\mathbf{d})$).

Let $d_{\min} = \min_{1\le j\le q}(d_j)$. We have

$$\sum_{k=0}^{k_f}\sum_{j=1}^{q}\boldsymbol{\eta}^T(k)P_j\boldsymbol{\eta}(k) = \sum_{l=0}^{k_f-d_{\min}}\sum_{j=1}^{q}\boldsymbol{\eta}^T(l)P_j\boldsymbol{\eta}(l) + \sum_{l=k_f-d_{\min}+1}^{k_f}\sum_{j=1}^{q}\boldsymbol{\eta}^T(l)P_j\boldsymbol{\eta}(l).$$

According to the initial conditions $\mathbf{x}(k) = 0$ and $\hat{\mathbf{x}}(k) = 0$ for $k = -d_{\max}, -d_{\max}+1, \ldots, -1$, where $d_{\max} = \max_{1\le j\le q}(d_j)$, it follows that

$$\begin{aligned}\sum_{k=0}^{k_f}\sum_{j=1}^{q}\boldsymbol{\eta}^T(k-d_j)P_j\boldsymbol{\eta}(k-d_j) &= \sum_{l=0}^{k_f-d_j}\sum_{j=1}^{q}\boldsymbol{\eta}^T(l)P_j\boldsymbol{\eta}(l)\\ &\le \sum_{l=0}^{k_f-d_{\min}}\sum_{j=1}^{q}\boldsymbol{\eta}^T(l)P_j\boldsymbol{\eta}(l)\end{aligned}$$

$$\begin{aligned}&\sum_{k=0}^{k_f}\sum_{j=1}^{q}[\boldsymbol{\eta}^T(k)P_j\boldsymbol{\eta}(k) - \boldsymbol{\eta}^T(k-d_j)P_j\boldsymbol{\eta}(k-d_j)]\\ &\quad\ge \sum_{k=0}^{k_f}\sum_{j=1}^{q}\boldsymbol{\eta}^T(k)P_j\boldsymbol{\eta}(k) - \sum_{l=0}^{k_f-d_{\min}}\sum_{j=1}^{q}\boldsymbol{\eta}^T(l)P_j\boldsymbol{\eta}(l)\\ &\quad= \sum_{l=k_f-d_{\min}+1}^{k_f}\sum_{j=1}^{q}\boldsymbol{\eta}^T(l)P_j\boldsymbol{\eta}(l)\\ &\quad\ge 0. \end{aligned}\tag{33}$$

By (33), (32) becomes

$$\begin{aligned}&\sum_{k=1}^{k_f}\mathbf{e}^T(k)Q\mathbf{e}(k)\\ &\quad\le \boldsymbol{\eta}^T(0)P\boldsymbol{\eta}(0) + \boldsymbol{\xi}^T(k_f,\mathbf{d})\\ &\qquad\times\left[\tilde{Q} - \Lambda^T P\Lambda + \Lambda^T\sum_{j=1}^{q}P_j\Lambda - \Upsilon^T S\Upsilon\right]\boldsymbol{\xi}(k_f,\mathbf{d})\\ &\qquad+ \sum_{k=0}^{k_f-1}\mathbf{e}^T(k)Q\mathbf{e}(k) + \sum_{k=0}^{k_f-1}\Delta V(k). \end{aligned}\tag{34}$$

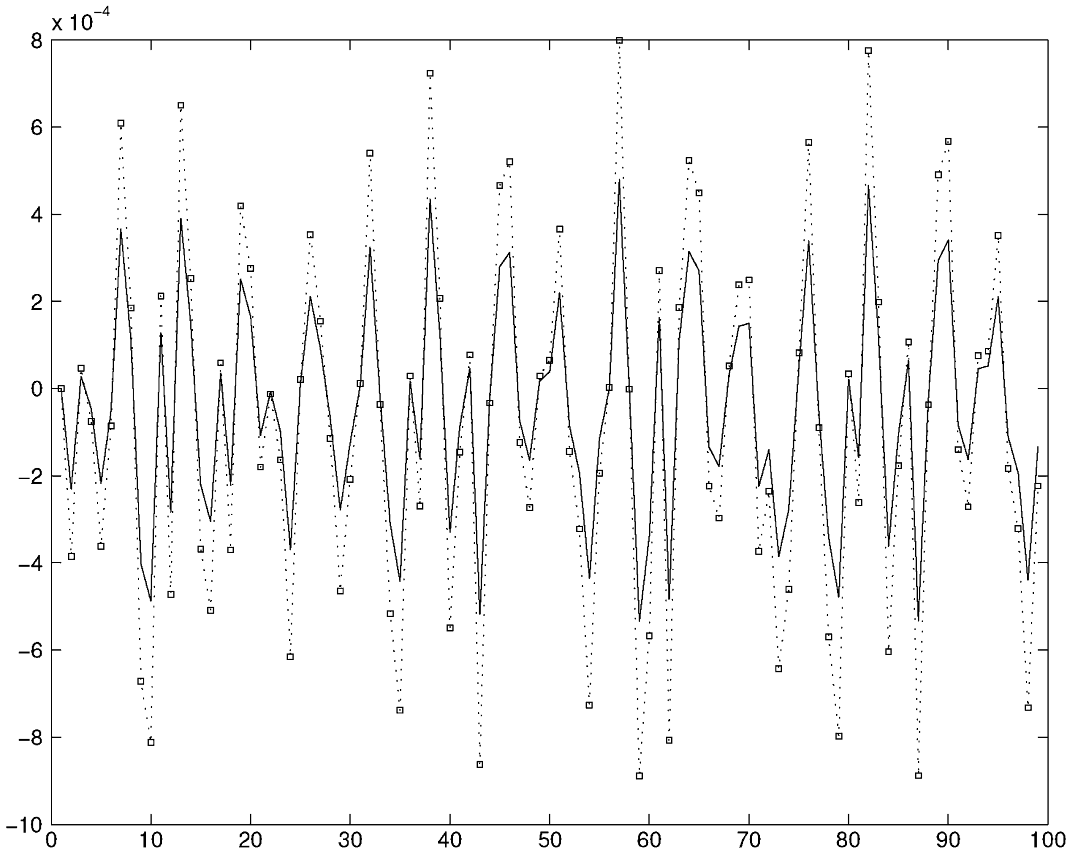

Fig. 2. The estimation error of $x_2(k) - \hat{x}_2(k)$ [the present method (solid line) and EKF (dash-dot line with square marker)].

Let $\tilde{Q} - \Lambda^T P\Lambda + \Lambda^T \sum_{j=1}^{q} P_j\Lambda - \Upsilon^T S\Upsilon < 0$. Then (34) can be rewritten as

$$\sum_{k=1}^{k_f} \mathbf{e}^T(k)Q\mathbf{e}(k) \le \boldsymbol{\eta}^T(0)P\boldsymbol{\eta}(0) + \sum_{k=0}^{k_f-1} \mathbf{e}^T(k)Q\mathbf{e}(k) + \sum_{k=0}^{k_f-1} \Delta V(k). \quad (35)$$

Substituting (30) into (35), it follows that

$$\begin{aligned}\sum_{k=1}^{k_f} \mathbf{e}^T(k)Q\mathbf{e}(k) &\le \boldsymbol{\eta}^T(0)P\boldsymbol{\eta}(0) + \gamma^2 \sum_{k=0}^{k_f-1} [\mathbf{w}^T(k)W_w\mathbf{w}(k) + \mathbf{v}^T(k)W_v\mathbf{v}(k)] \\ &= \boldsymbol{\xi}^T(0,\mathbf{d})\Lambda^T P\Lambda\boldsymbol{\xi}(0,\mathbf{d}) + \gamma^2 \sum_{k=0}^{k_f-1} [\mathbf{w}^T(k)W_w\mathbf{w}(k) + \mathbf{v}^T(k)W_v\mathbf{v}(k)] \\ &= \gamma^2 \left[\boldsymbol{\xi}^T(0,\mathbf{d})\tilde{P}_0\boldsymbol{\xi}(0,\mathbf{d}) + \sum_{k=0}^{k_f-1} (\mathbf{w}^T(k)W_w\mathbf{w}(k) + \mathbf{v}^T(k)W_v\mathbf{v}(k))\right]\end{aligned} \quad (36)$$

where $\tilde{P}_0 = (1/\gamma^2)\Lambda^T P\Lambda$.

The assumption

$$\tilde{Q} - \Lambda^T P\Lambda + \Lambda^T \sum_{j=1}^{q} P_j\Lambda - \Upsilon^T S\Upsilon < 0$$

can equivalently be written as

$$\begin{aligned}&\begin{bmatrix} \tilde{Q}_{11} - P + \sum_{j=1}^{q} P_j & \tilde{Q}_{12} \\ \tilde{Q}_{21} & \tilde{Q}_{22} - S \end{bmatrix} = \tilde{Q} + \begin{bmatrix} -P + \sum_{j=1}^{q} P_j & 0 \\ 0 & -S \end{bmatrix} \\ &= \begin{bmatrix} \tilde{L}^T \\ \tilde{L}_{d1}^T \\ \vdots \\ \tilde{L}_{dq}^T \end{bmatrix} Q[\tilde{L} \quad \tilde{L}_{d1} \quad \dots \quad \tilde{L}_{dq}] + \begin{bmatrix} -P + \sum_{j=1}^{q} P_j & 0 \\ 0 & -S \end{bmatrix} \\ &< 0.\end{aligned} \quad (37)$$

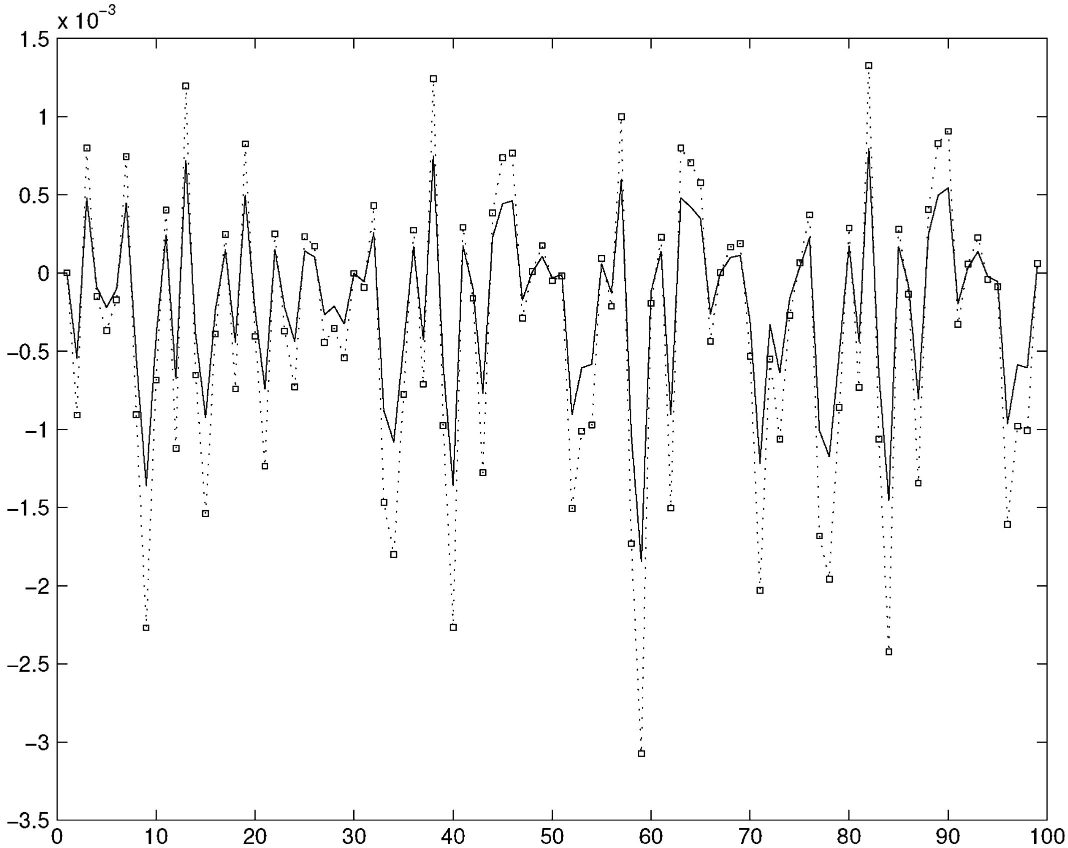

Fig. 3. The estimation error of $s(k) - \hat{s}(k)$ [the present method (solid line) and EKF (dash-dot line with square marker)].

According to Schur complements in Lemma 3.1, (37) is equivalent to

$$\begin{bmatrix} -P + \sum_{j=1}^{q} P_j & 0 & \tilde{L^T} \\ 0 & -S & \tilde{L_d}^T \\ \tilde{L} & \tilde{L}_d & -Q^{-1} \end{bmatrix} < 0. \tag{38}$$

Comparing $-P + \sum_{j=1}^{q} P_j + \tau\Theta < 0$ with $-P + \sum_{j=1}^{q} P_j < 0$, and $-S + \tau\Theta_d < 0$ with $-S < 0$, it is clear that any solution to the former will also satisfy the latter due to the fact that $\tau\Theta > 0$ and $\tau\Theta_d > 0$. Therefore, according to Schur complements, the LMI of (22) ensures that (38) holds.

Hence, the H_∞ index is achieved with a prescribed γ^2, as shown in (36). This completes the proof. ■

From the definitions of Θ and Θ_d given in the theorem, we can see that it would be more difficult to obtain a negative definite matrix in (22) if the norms of $\Omega + \Psi$ and $\Omega_{dj} + \Psi_{dj}$ are very large, as a result of large modeling errors.

IV. Fuzzy H_∞ Filter Design

Theorem 3.1 provides a sufficient condition for the existence of fuzzy H_∞ filter assuring an H_∞ performance for system in (1). In order to obtain a fuzzy H_∞ filter, the most important work is to solve $P = P^T$ and $P_j = P_j^T$ from (22).

Next, we design a fuzzy H_∞ filter according to Theorem 3.1. We obtain the following theorem.

Theorem 4.1: The condition (22) of Theorem 3.1 for the existence of an admissible H_∞ filter is equivalent to (39) and (40) shown at the bottom of the next page, where $*$ denotes the corresponding symmetric element

$$\bar{\Lambda}_d = \text{diag}\left\{\bar{\Lambda}_1^d, \ldots, \bar{\Lambda}_q^d\right\}$$
$$\bar{\Lambda}_j^d = \begin{bmatrix} -\bar{Z}_{11}^j + \tau(\Omega_{dj} + \Psi_{dj}) & -\bar{Z}_{12}^j + \tau(\Omega_{dj} + \Psi_{dj}) \\ -(\bar{Z}_{12}^j)^T + \tau(\Omega_{dj} + \Psi_{dj}) & -\bar{Z}_{22}^j + \tau(\Omega_{dj} + \Psi_{dj}) \end{bmatrix}$$

$j = 1, 2, \ldots, q$

$$\begin{aligned} \bar{\Psi}_{11}^d &= \left[\bar{R}A_{1i}^d, \bar{R}A_{1i}^d, \ldots, \bar{R}A_{qi}^d, \bar{R}A_{qi}^d\right], \\ \bar{\Psi}_{12}^d &= \left[XA_{1i}^d + \bar{K}_l C_{1i}^d + M_{1l}^d, XA_{1i}^d + \bar{K}_l C_{1i}^d, \ldots,\right. \\ &\qquad \left. XA_{qi}^d + \bar{K}_l C_{qi}^d + M_{ql}^d, XA_{qi}^d + \bar{K}_l C_{qi}^d\right], \\ \bar{\Psi}_2^d &= [L_{d1} - L_{d1}U, L_{d1}, \ldots, L_{dq} - L_{dq}U, L_{dq}], \\ \bar{Z}_j &= \begin{bmatrix} \bar{Z}_{11}^j & \bar{Z}_{12}^j \\ \left(\bar{Z}_{12}^j\right)^T & \bar{Z}_{22}^j \end{bmatrix}, \quad j = 1, 2, \ldots, q \end{aligned}$$

$X \in R^{n\times n}$, $\bar{R} \in R^{n\times n}$, and $\bar{Z}_j \in R^{n\times n}$ are positive definite matrices, and $M_l \in R^{n\times n}, M_{jl}^d \in R^{n\times n}$, $U \in R^{n\times n}$, and $\bar{K}_l \in R^{n\times m}$.

In addition, an admissible filter is given by

$$K_l = U(\bar{R} - X)^{-1}\bar{K}_l.$$

Proof:

a) Sufficiency: Assume that (22) is true. According to $\sum_{i=1}^{N} h_i(\mathbf{x}(k)) = 1$, $\sum_{l=1}^{N} h_l(\mathbf{x}(k)) = 1$, and

$\sum_{m=1}^{N} h_m(\mathbf{x}(k)) = 1$, (22) can be written as shown in (41) at the bottom of the page, for $i, l, s = 1, 2, \ldots, N$.

According to (16), we have

Let P, P^{-1}, and P_j $(j = 1, 2, \ldots, q)$ be partitioned according to $\boldsymbol{\eta}^T = [\mathbf{x}\ \hat{\mathbf{x}}]^T$ as

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix},$$
$$P_j = \begin{bmatrix} Z_{11}^j & Z_{12}^j \\ (Z_{12}^j)^T & Z_{22}^j \end{bmatrix}. \tag{43}$$

Since $P = P^T \in R^{2n \times 2n}$, such a partition renders that P_{12} and S_{12}^T are both square matrices. Assume that S_{12} is nonsingular. Define the following matrix:

$$J = \begin{bmatrix} S_{11} & I \\ S_{12}^T & 0 \end{bmatrix} \begin{bmatrix} S_{11}^{-1} & 0 \\ 0 & I \end{bmatrix} = \begin{bmatrix} I & I \\ S_{12}^T S_{11}^{-1} & 0 \end{bmatrix}. \tag{44}$$

Then J is nonsingular. Premultiplying (42), as shown at the bottom of the page, by $\operatorname{diag}\{J^T, \underbrace{J^T, \ldots, J^T}_{q}, I, I, I, J^T, I\}$ and postmultiplying by $\operatorname{diag}\{J, \underbrace{J, \ldots, J}_{q}, I, I, I, J, I\}$,

$$\begin{bmatrix}
-\bar{R} + \sum_{j=1}^{q} \bar{Z}_{11}^j + \tau(\Omega + \Psi) & * & * & * & * & * & * & * & * & * & * & * \\
-\bar{R} + \sum_{j=1}^{q} (\bar{Z}_{12}^j)^T + \tau(\Omega + \Psi) & -X + \sum_{j=1}^{q} \bar{Z}_{22}^j + \tau(\Omega + \Psi) & * & * & * & * & * & * & * & * & * & * \\
0 & 0 & \bar{\Lambda}_d & * & * & * & * & * & * & * & * & * \\
0 & 0 & 0 & -\tau I & * & * & * & * & * & * & * & * \\
0 & 0 & 0 & 0 & -\tau I & * & * & * & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & -\tau I & * & * & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & -\tau I & * & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2 W_w & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2 W_v & * & * & * \\
\bar{R}A_i & \bar{R}A_i & \bar{\Psi}_{11}^d & \bar{R} & 0 & \bar{R} & 0 & \bar{R}B & 0 & -\bar{R} & * & * \\
XA_i + \bar{K}_l C_i + M_l & XA_i + \bar{K}_l C_i & \bar{\Psi}_{12}^d & X & \bar{K}_l & X & \bar{K}_l & XB & \bar{K}_l G & -\bar{R} & -X & * \\
L - LU & L & \bar{\Psi}_2^d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Q^{-1}
\end{bmatrix} < 0 \tag{39}$$

$$X - \bar{R} > 0 \tag{40}$$

$$\begin{bmatrix}
-P + \sum_{j=1}^{q} P_j + \tau\Theta & * & * & * & * & * & * \\
0 & -S + \tau\Theta_d & * & * & * & * & * \\
0 & 0 & -\tau I & * & * & * & * \\
0 & 0 & 0 & -\tau I & * & * & * \\
0 & 0 & 0 & 0 & -\gamma^2 W & * & * \\
P\tilde{A}_{ils} & P\left[\tilde{A}_{1ils}^d, \ldots, \tilde{A}_{qils}^d\right] & P\tilde{E}_l & P\tilde{E}_l^d & P\tilde{G}_l & -P & * \\
[L, \quad -L] & [L_{d1}, \quad -L_{d1}, \ldots, L_{dq}, \quad -L_{dq}] & 0 & 0 & 0 & 0 & -Q^{-1}
\end{bmatrix} < 0 \tag{41}$$

$$\begin{bmatrix}
-P + \sum_{j=1}^{q} P_j + \tau\Theta & * & \cdots & * & * & * & * & * & * \\
0 & -P_1 + \tau\Theta_{d1} & \cdots & * & * & * & * & * & * \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & -P_q + \tau\Theta_{dq} & * & * & * & * & * \\
0 & 0 & \cdots & 0 & -\tau I & * & * & * & * \\
0 & 0 & \cdots & 0 & 0 & -\tau I & * & * & * \\
0 & 0 & \cdots & 0 & 0 & 0 & -\gamma^2 W & * & * \\
P\begin{bmatrix} A_i & 0 \\ K_l C_i A_l - K_l C_s \end{bmatrix} & P\begin{bmatrix} A_{1i}^d & 0 \\ K_l C_{1i}^d A_{1l}^d - K_l C_{1s}^d \end{bmatrix} & \cdots & P\begin{bmatrix} A_{qi}^d & 0 \\ K_l C_{qi}^d A_{ql}^d - K_l C_{qs}^d \end{bmatrix} & P\begin{bmatrix} I & 0 \\ 0 & K_l \end{bmatrix} & P\begin{bmatrix} I & 0 \\ 0 & K_l \end{bmatrix} & P\begin{bmatrix} B & 0 \\ 0 & K_l G \end{bmatrix} & -P & * \\
[L \quad -L] & [L_{d1} \quad -L_{d1}] & \cdots & [L_{dq} \quad -L_{dq}] & 0 & 0 & 0 & 0 & -Q^{-1}
\end{bmatrix} < 0 \tag{42}$$

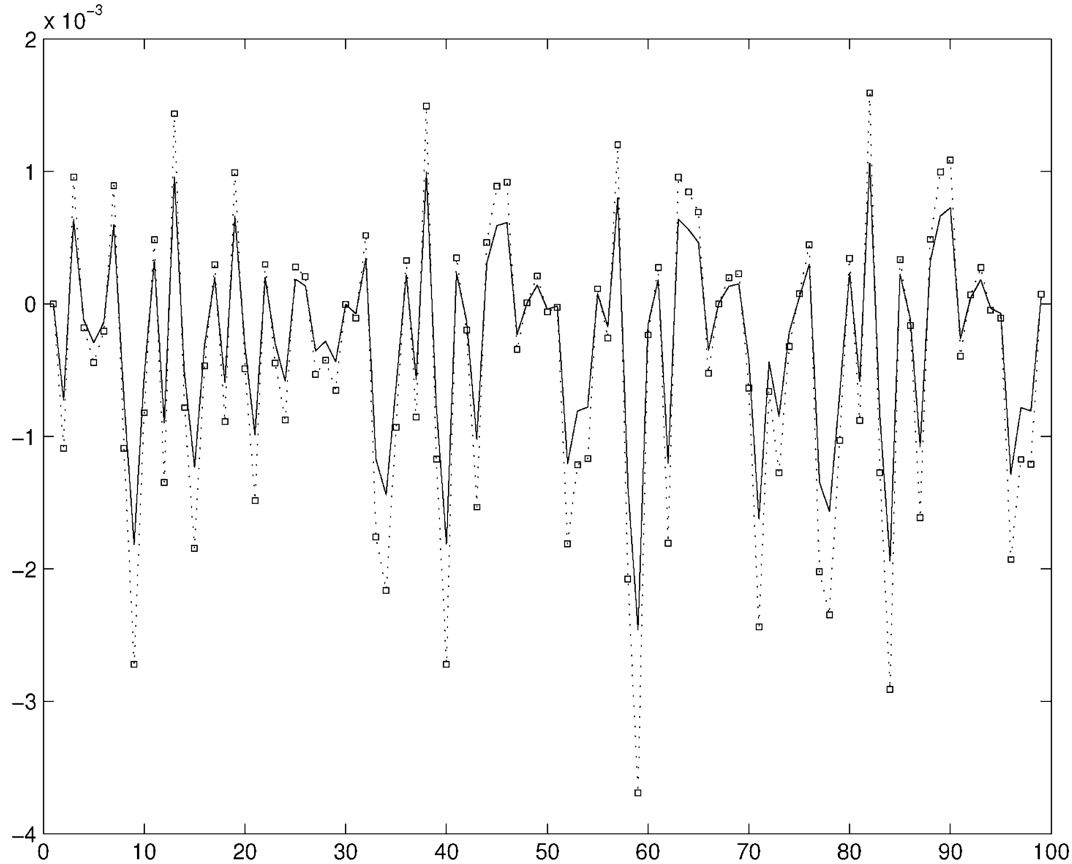

Fig. 4. The estimation error of $s(k) - \hat{s}(k)$ [the present method (solid line) and EKF (dash-dot line with square marker)].

and defining $X = P_{11}$, $U = S_{12}^T\bar{R}$, $\bar{K}_l = P_{12}K_l$, $\bar{R} = S_{11}^{-1}$, $M_l = P_{12}A_lS_{12}^T\bar{R} - P_{12}K_lC_sS_{12}^T\bar{R}$, $M_{jl}^d = P_{12}A_{jl}^dS_{12}^T\bar{R} - P_{12}K_lC_{js}^dS_{12}^T\bar{R}$, and

$$\begin{aligned}\bar{Z}_j &= \begin{bmatrix} \bar{Z}_{11}^j & \bar{Z}_{12}^j \\ \left(\bar{Z}_{12}^j\right)^T & \bar{Z}_{22}^j \end{bmatrix} \\ &= \begin{bmatrix} S_{11}^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ I & 0 \end{bmatrix} \begin{bmatrix} Z_{11}^j & Z_{12}^j \\ \left(Z_{12}^j\right)^T & Z_{22}^j \end{bmatrix} \\ &\quad \times \begin{bmatrix} S_{11} & I \\ S_{12}^T & 0 \end{bmatrix} \begin{bmatrix} S_{11}^{-1} & 0 \\ 0 & I \end{bmatrix}, \; j = 1, 2, \ldots, q\end{aligned}$$

we obtain (39).

Since $\bar{K}_l = P_{12}K_l$, then it yields

$$K_l = P_{12}^{-1}\bar{K}_l. \tag{45}$$

Since $PP^{-1} = I$, we have

$$I - P_{11}S_{11} = P_{12}S_{12}^T. \tag{46}$$

Equation (46) can be rewritten as

$$I - X\bar{R}^{-1} = P_{12}S_{12}^T. \tag{47}$$

According to $U = S_{12}^TS_{11}^{-1} = S_{12}^T\bar{R}$, we can obtain the following equation from (45) and (47):

$$K_l = P_{12}^{-1}\bar{K}_l = U(\bar{R} - X)^{-1}\bar{K}_l. \tag{48}$$

In addition, $P > 0$ holds if and only if $J^TPJ = \begin{bmatrix} S_{11} & I \\ I & P_{11} \end{bmatrix} > 0$ holds, which is equivalent to

$$P_{11} - S_{11}^{-1} = X - \bar{R} > 0. \tag{49}$$

b) Necessity: Suppose there exist matrices X, $\bar{R}, \bar{Z}_j$, M_l, M_{jl}^d, U, and $\bar{K}_l$ satisfying (39). Since (40) implies that $\bar{R} - X$ is nonsingular, we can construct a filter as $K_l = U(\bar{R} - X)^{-1}\bar{K}_l$. Then, premultiplying (39) by $\Upsilon_2 = \text{diag}[(J^T)^{-1}, \underbrace{(J^T)^{-1}, \ldots, (J^T)^{-1}}_{q}, I, I, I, (J^T)^{-1}, I]$ and postmultiplying by Υ_2^T, we obtain LMIs of (42) and then (22) can be obtained. Thus, LMIs (39) and (40) are equivalent to (22). Therefore, an admissible filter can be given by $K_l = U(\bar{R} - X)^{-1}\bar{K}_l$.

This completes the proof. ■

Remark 4.1: In view of Theorem 4.1, the fuzzy H_∞ filtering problem (15) can be solved by the feasibility problem of the LMIs (39). In fact, any feasible solution to (39) yields a suitable filter. To obtain a better filtering performance against disturbances, the attenuation level γ^2 can be reduced to the min-

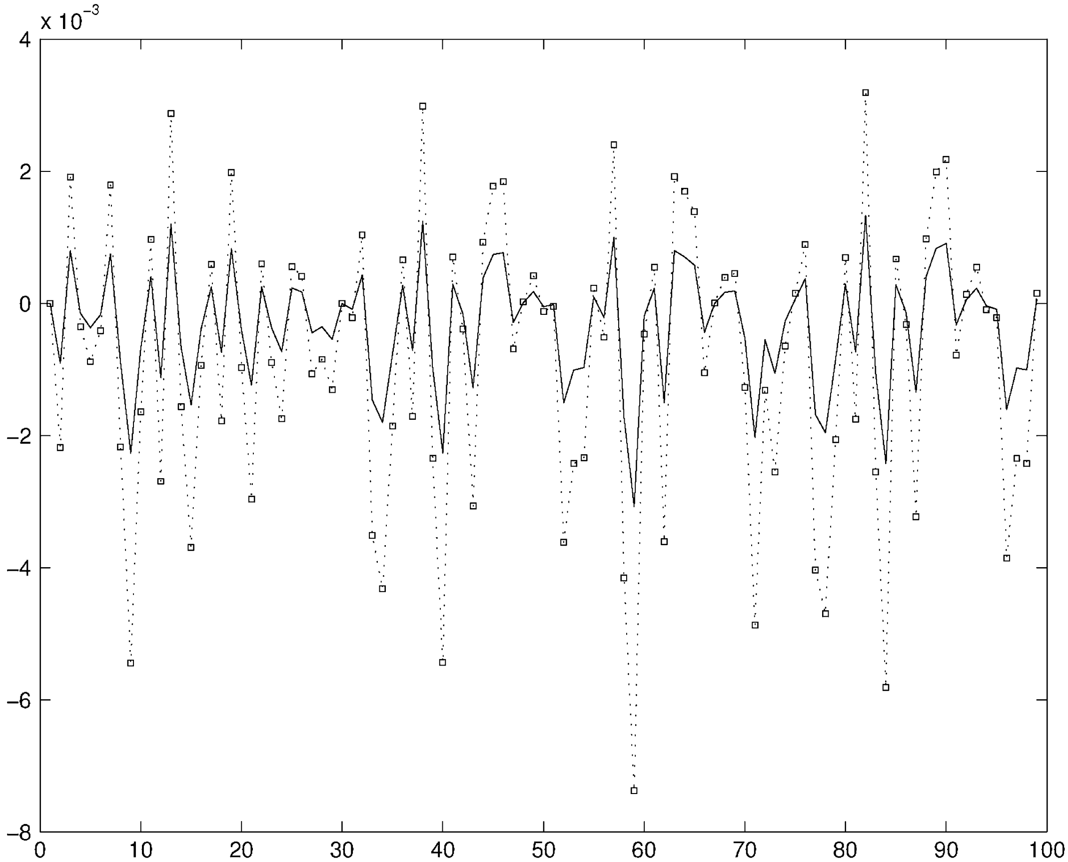

Fig. 5. The estimation error of $s(k) - \hat{s}(k)$ [the present method (solid line) and EKF (dash-dot line with square marker)].

imum possible value such that (19) is satisfied. The H_∞ filter with the smallest γ^2 attenuation level obtained from Theorem 4.1 can be determined by solving the following convex optimization problem:

$$\min_{U,\bar{R},X,\bar{K}_l} \delta \text{ subject to (39) with } \gamma^2 = \delta. \tag{50}$$

■

The design procedure of fuzzy H_∞ filter is summarized as follows.

Step 1) Construct the T–S fuzzy model (2) for the nonlinear system (1).

Step 2) Given $\Omega, \Omega_d, \Psi, \Psi_d$, Q, and an initial attenuation level γ^2.

Step 3) Solve the LMI problem (39) to obtain U, $\bar{R}$, X, $\bar{K}_l$. We obtain K_l according to (48).

Step 4) Decrease the attenuation level γ^2, and repeat Steps 3) and 4) until U, $\bar{R}$, X, $\bar{K}_l$ are obtained with the smallest possible γ^2;

Step 5) Obtain the fuzzy H_∞ filter as in (15).

■

There are two main approaches in fuzzy modeling. One is fuzzy model identification using input–output data [26], [32], [33]. The other is fuzzy model construction (fuzzy IF-THEN rules) using sector nonlinearity or local approximation for the case where a mathematical model of the nonlinear system is available [22]. We summarize in the next remark the modeling method using input–output data pairs.

Remark 4.2: The distribution of input–output data pairs should be sufficient and proper. In order to identify a fuzzy model for (1), we assume $\mathbf{w}(k)$ and $\mathbf{v}(k)$ are independent identically distributed random signals uniformly distributed in the interval [−1,+1] [26]. Then A_i, A_{ji}^d, C_i, and C_{ji}^d in the fuzzy rules [see (2)] can be obtained using the following steps.

Step 1) Specify the number of fuzzy sets and membership functions for the premise variables (i.e., part or all of state variables) in advance.

Step 2) Use recursive least square algorithm to obtain the parameters of T–S fuzzy model.

■

Remark 4.3: Ω, Ω_d, Ψ, and Ψ_d can be estimated by offline simulations. We provide a method for estimating Ω, Ω_d, Ψ, and Ψ_d as follows. We choose

$$\alpha = \max_{\mathbf{x}(k)\neq 0} \left\{ \frac{\Delta F^T(\mathbf{x}(k))\Delta F(\mathbf{x}(k))}{\mathbf{x}^T(k)\mathbf{x}(k)} \right\}$$

$$\alpha_d = \max_{\mathbf{x}(k)\neq 0} \left\{ \frac{\Delta F_d^T(\mathbf{x}(k,\mathbf{d},q))\Delta F_d(\mathbf{x}(k,\mathbf{d},q))}{\sum_{j=1}^{q} \mathbf{x}^T(k-d_j)\mathbf{x}(k-d_j)} \right\}$$

$$\beta = \max_{\mathbf{x}(k)\neq 0} \left\{ \frac{\Delta H^T(\mathbf{x}(k))\Delta H(\mathbf{x}(k))}{\mathbf{x}^T(k)\mathbf{x}(k)} \right\}$$

$$\beta_d = \max_{\mathbf{x}(k)\neq 0} \left\{ \frac{\Delta H_d^T(\mathbf{x}(k,\mathbf{d},q))\Delta H_d(\mathbf{x}(k,\mathbf{d},q))}{\sum_{j=1}^{q} \mathbf{x}^T(k-d_j)\mathbf{x}(k-d_j)} \right\}.$$

We can choose Ω, Ω_d, Ψ, and Ψ_d as

$$\begin{aligned}\Omega &= \text{diag}(\alpha, \ldots, \alpha), \quad \Omega_d = \text{diag}(\Omega_{d1}, \Omega_{d2}, \ldots, \Omega_{dq})\\ \Psi &= \text{diag}(\beta, \ldots, \beta), \quad \Psi_d = \text{diag}(\Psi_{d1}, \Psi_{d2}, \ldots, \Psi_{dq})\end{aligned} \tag{51}$$

where $\Omega_{d1} = \Omega_{d2} = \cdots = \Omega_{dq} = \text{diag}(\alpha_d, \ldots, \alpha_d)$, and $\Psi_{d1} = \Psi_{d2} = \cdots = \Psi_{dq} = \text{diag}(\beta_d, \ldots, \beta_d)$. In the above calculations, we choose a set of values for $\mathbf{x}(0)$. From each $\mathbf{x}(0)$, we generate two trajectories according to (1) and (2). We can then obtain ΔF, ΔF_d, ΔH, and ΔH_d according to (5)–(8). ■

V. Simulation Example

Consider the following discrete-time nonlinear system with multiple time delays:

$$\begin{aligned}x_1(k+1) &= 0.24x_1(k) - 0.13(x_2(k) - x_1^2(k))\\ &\quad + 0.18x_1(k-d_1) + 0.38x_2(k-d_2)\\ &\quad + 0.1x_1^2(k-d_1) + w_1(k)\\ x_2(k+1) &= 0.13x_1(k) + 0.24(x_2(k) - x_1^2(k))\\ &\quad + 0.26x_1(k-d_1) + 0.15x_2(k-d_2)\\ &\quad + 0.1x_1^2(k-d_1) + w_2(k)\\ y(k) &= x_1(k) + 0.1x_1^2(k) + x_2(k) + 0.1x_2^2(k)\\ &\quad + 0.05x_1(k-d_1) + 0.05x_2(k-d_2)\\ &\quad + 0.025x_1^2(k-d_1) + 0.025x_2^2(k-d_2)\\ &\quad + v(k)\\ s(k) &= x_1(k) + 2x_2(k).\end{aligned} \tag{52}$$

It is assumed that the initial conditions are

$$[x_1(0), x_2(0), \hat{x}_1(0), \hat{x}_2(0)] = [0.1, -0.1, 0, 0.1].$$

We assume that x_1 and $x_2 \in [-1, 1]$. Let $\mathbf{d} = [d_1, d_2]$. Assume that $d_1 = 1$ and $d_2 = 2$. The design purpose is to construct fuzzy H_∞ filter to estimate $s(k)$.

The fuzzy H_∞ filter design procedure for system (52) is as follows.

Step 1: We use the local sector nonlinearity method to obtain fuzzy model [22]. The number of fuzzy sets for the state variables $x_1(k)$ and $x_2(k)$ is two: large (close to 1) and small (close to -1). The membership functions are calculated as

$$M_1(x(k)) = \frac{1}{2}(1 + x(k)), \quad M_2(x(k)) = \frac{1}{2}(1 - x(k)) \tag{53}$$

where $x(k)$ denotes $x_1(k)$ and $x_2(k)$, respectively. When x_1 is close to 1 (-1), we have $x_1^2 = x_1$ ($x_1^2 = -x_1$). In this case, we replace x_1^2 by x_1 (x_1^2 by $-x_1$). Similar arguments can be applied to x_2. We then obtain the corresponding local linear approximation model in fuzzy partition spaces to obtain matrices A_i A_{ji}^d, C_i, and C_{ji}^d, $i = 1, \ldots, 4$, $j = 1, 2$.

The T–S fuzzy model based on the linear local models for system (52) is as follows.

Rule 1: IF x_1 is about 1 and x_2 is about 1, THEN

$$\begin{aligned}\mathbf{x}(k+1) &= A_1\mathbf{x}(k) + \sum_{j=1}^{2} A_{j1}^d \mathbf{x}(k-d_j) + B\mathbf{w}(k)\\ y(k) &= C_1\mathbf{x}(k) + \sum_{j=1}^{q} C_{j1}^d \mathbf{x}(k-d_j) + G\mathbf{v}(k).\end{aligned}$$

Rule 2: IF x_1 is about 1 and x_2 is about -1, THEN

$$\begin{aligned}\mathbf{x}(k+1) &= A_2\mathbf{x}(k) + \sum_{j=1}^{2} A_{j2}^d \mathbf{x}(k-d_j) + B\mathbf{w}(k)\\ y(k) &= C_2\mathbf{x}(k) + \sum_{j=1}^{q} C_{j2}^d \mathbf{x}(k-d_j) + G\mathbf{v}(k).\end{aligned}$$

Rule 3: IF x_1 is about -1 and x_2 is about 1, THEN

$$\begin{aligned}\mathbf{x}(k+1) &= A_3\mathbf{x}(k) + \sum_{j=1}^{2} A_{j3}^d \mathbf{x}(k-d_j) + B\mathbf{w}(k)\\ y(k) &= C_3\mathbf{x}(k) + \sum_{j=1}^{q} C_{j3}^d \mathbf{x}(k-d_j) + G\mathbf{v}(k).\end{aligned}$$

Rule 4: IF x_1 is about -1 and x_2 is about -1, THEN

$$\begin{aligned}\mathbf{x}(k+1) &= A_4\mathbf{x}(k) + \sum_{j=1}^{2} A_{j4}^d \mathbf{x}(k-d_j) + B\mathbf{w}(k)\\ y(k) &= C_4\mathbf{x}(k) + \sum_{j=1}^{q} C_{j4}^d \mathbf{x}(k-d_j) + G\mathbf{v}(k)\end{aligned}$$

where

$$\begin{aligned}A_1 &= \begin{bmatrix} 0.37 & -0.13 \\ -0.11 & 0.24 \end{bmatrix}, \quad A_2 = A_1,\\ A_3 &= \begin{bmatrix} 0.11 & -0.13 \\ 0.37 & 0.24 \end{bmatrix}, \quad A_4 = A_3,\\ A_{11}^d &= \begin{bmatrix} 0.28 & 0 \\ 0.36 & 0 \end{bmatrix}, \quad A_{21}^d = \begin{bmatrix} 0 & 0.38 \\ 0 & 0.15 \end{bmatrix},\\ A_{12}^d &= \begin{bmatrix} 0.28 & 0 \\ 0.36 & 0 \end{bmatrix}, \quad A_{22}^d = \begin{bmatrix} 0 & 0.38 \\ 0 & 0.15 \end{bmatrix},\\ A_{13}^d &= \begin{bmatrix} 0.08 & 0 \\ 0.16 & 0 \end{bmatrix}, \quad A_{23}^d = \begin{bmatrix} 0 & 0.38 \\ 0 & 0.15 \end{bmatrix},\\ A_{14}^d &= \begin{bmatrix} 0.08 & 0 \\ 0.16 & 0 \end{bmatrix}, \quad A_{24}^d = \begin{bmatrix} 0 & 0.38 \\ 0 & 0.15 \end{bmatrix},\\ C_1 &= [1.1 \quad 1.1], \quad C_2 = [1.1 \quad 0.9],\\ C_3 &= [0.9 \quad 1.1], \quad C_4 = [0.9 \quad 0.9],\\ C_{11}^d &= [0.075 \quad 0], \quad C_{21}^d = [0 \quad 0.075],\\ C_{12}^d &= [0.075 \quad 0], \quad C_{22}^d = [0 \quad 0.025],\\ C_{13}^d &= [0.025 \quad 0], \quad C_{23}^d = [0 \quad 0.075],\\ C_{14}^d &= [0.025 \quad 0], \quad C_{24}^d = [0 \quad 0.025],\\ B &= 0.1I, \quad G = 0.1I.\end{aligned}$$

Step 2: We select 4000 input–output data pairs from (52) and 4000 input–output data pairs from the fuzzy model obtained, respectively. For all input–output data pairs, we compute the corresponding approximation error terms. We obtain $\Omega = 8.6673 \times 10^{-4} I$, $\Omega_d = 1.9903 \times 10^{-5} I$, $\Psi = 1.87 \times 10^{-5} I$, and $\Psi_d = 1.5823 \times 10^{-4} I$ according to Remark 4.3.

Step 3: Solve the LMI problem (39). We obtain $\gamma^2 = 0.9282$ and the fuzzy estimation gains K_l as follows:

$$K_1 = \begin{bmatrix} 0.125248 \\ 0.061572 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.135008 \\ 0.066347 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} 0.061736 \\ 0.034974 \end{bmatrix}, \quad K_4 = \begin{bmatrix} 0.065769 \\ 0.037388 \end{bmatrix}.$$

Step 4: Construct the fuzzy H_∞ filter as follows:

$$\hat{\mathbf{x}}(k+1) = \sum_{l=1}^{4} h_l(\hat{\mathbf{x}}(k))(A_l \hat{\mathbf{x}}(k) + \sum_{j=1}^{q} A_{jl}^d \hat{\mathbf{x}}(k-d_j) + K_l(y(k) - \hat{y}(k))).$$

Figs. 1 and 2 show the estimation errors for $x_1(k)$ and $x_2(k)$, respectively, by the present fuzzy H_∞ filter and the extended Kalman filter (EKF), where we assume that $w_1(k) = 0.1\sin(0.2\pi k)$, $w_2(k) = 0.1\cos(0.2\pi k)$, and the noise $v(k)$ is normally distributed with zero mean and variance 0.01. Fig. 3 shows the estimation error for $s(k)$ by the present fuzzy H_∞ filter and the EKF. Fig. 4 shows the estimation errors for $s(k)$ when the noises $w_1(k)$ and $w_2(k)$ are normally distributed with zero mean and variance 0.01. Fig. 5 shows the estimation errors for $s(k)$ when the noise $w_1(k)$ and $v(k)$ are normally distributed with zero mean and variance 0.01 and $w_2(k) = 0.5w_2(k-1) + w_1(k)$ is a colored noise. Dash-dot lines with square markers denote the estimation errors using the EKF under the statistical conditions $E[w(k)w(k)^T] = 0.003I$, $E[v(k)v(k)^T] = 0.01$. From the above simulation results, we see that the present fuzzy H_∞ filter can obtain better estimation and more robust performance than the EKF. The proposed fuzzy H_∞ filter is obtained without any information about external disturbances and measurement noise, as long as they are bounded. In fact, the statistical properties of $\mathbf{w}(k)$ and $\mathbf{v}(k)$ change with time and are rarely known beforehand. Therefore, the proposed fuzzy H_∞ filter is more robust than the EKF.

VI. Conclusion

In this paper, based on the T–S fuzzy model, a fuzzy H_∞ filter is designed for a class of nonlinear discrete-time systems with multiple time delays.

The advantages of the present fuzzy H_∞ filter over the Kalman filter are as follows.

1) No statistical assumption on the external disturbances and measurement noise is needed.
2) The proposed fuzzy H_∞ filter for the nonlinear system can tolerate approximation errors based on the model error bounds, which can be regarded as the worst case approximation error.
3) Fuzzy H_∞ filters are more robust than the Kalman filter in the case of uncertain external disturbances and measurement noise.
4) The problem of fuzzy H_∞ filter design is converted into a linear matrix inequality problem that can efficiently be solved using convex optimization techniques, such as the interior point algorithm.

It should be noticed that the fuzzy H_∞ filtering method of Theorem 3.1 can be used in a number of important problems in signal processing, where delays are unavoidable and must be taken into account in a realistic filter design such as echo cancellation, local loop equalization, multipath propagation in mobile communication, array signal processing, and congestion analysis and control in high-speed communication networks [17].

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