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Finite-horizon neuro-optimal tracking control for a class of discrete-time nonlinear systems using adaptive dynamic programming approach ☆

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ABSTRACT

In this paper, a finite-horizon neuro-optimal tracking control strategy for a class of discrete-time nonlinear systems is proposed. Through system transformation, the optimal tracking problem is converted into designing a finite-horizon optimal regulator for the tracking error dynamics. Then, with convergence analysis in terms of cost function and control law, the iterative adaptive dynamic programming (ADP) algorithm via heuristic dynamic programming (HDP) technique is introduced to obtain the finite-horizon optimal tracking controller which makes the cost function close to its optimal value within an ε-error bound. Three neural networks are used as parametric structures to implement the algorithm, which aims at approximating the cost function, the control law, and the error dynamics, respectively. Two simulation examples are included to complement the theoretical discussions.

1. Introduction

It is well known that the optimal tracking control problem has been the focus of control systems community for several decades since it is usually encountered in real world systems [1–3]. In the case of infinite-horizon optimal tracking control, the system will not be tracked until the time reaches infinity, while for the finite case, the system must be tracked to a reference trajectory in a finite duration of time. Since many limitations exist in traditional optimal tracking control approaches, such as plant inversion [2] and linearization [3], it is necessary to design direct optimal tracking control schemes for nonlinear systems. In this paper, we will study how to solve this problem through the framework of Hamilton–Jacobi–Bellman (HJB) [4] equation from optimal control theory. Unlike the open-loop optimal controller design for nonlinear systems, however, for closed-loop optimal feedback control, it is difficult to solve directly the time-varying HJB equation which involves solving either nonlinear partial difference or differential equations. Though dynamic programming (DP) has been an useful computational technique in solving optimal control problems for many years, it is often computationally untenable to run it to obtain the optimal solution due to the "curse of dimensionality" [5].

As Poggio and Girosi [6] stated, the problem of learning between input and output spaces is equivalent to that of synthesizing an associative memory that retrieves appropriate output when the input is present and generalizes when a new input is applied. With strong capabilities of self-learning and adaptivity, artificial neural networks (ANN or NN) are an effective tool for implementing intelligent control [7–10]. Besides, it has been used for universal function approximation in adaptive/approximate dynamic programming (ADP) algorithms, which were proposed in [9–11] as a method for solving optimal control problems forward-in-time. There are several synonyms used for ADP including "adaptive dynamic programming" [12–14], "approximate dynamic programming" [9,15,16], "neuro-dynamic programming" [17], "neural dynamic programming" (NDP) [18], "adaptive critic designs" [19], and "reinforcement learning" [15,20]. As an effective intelligent control method, ADP and the related research have gained much attention from researchers [9–19,21–35]. Very good surveys were given in Wang et al. [13], Lewis and Vrabie [14], and Balakrishnan et al. [25]. According to [9,19], ADP approaches were classified into several main schemes: heuristic dynamic programming (HDP), action-dependent HDP (ADHDP), also known as Q-learning [20], dual heuristic dynamic programming (DHP), ADDHP, globalized DHP (GDHP), and ADGDHP. Al-Tamimi et al. [16] proposed a greedy HDP algorithm to solve the discrete-time HJB (DTHJB) equation for optimal control of nonlinear systems. Wang et al. [23] developed an ε-ADP algorithm for studying finite-horizon optimal control of discrete-time nonlinear systems.

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With the rapid development of NN technology and recently, the ADP method, various new strategies were devised to deal with the optimal tracking control problems. Park et al. [36] used the multilayer NN to design an optimal tracking neuro-controller for discrete-time nonlinear systems with quadratic cost function. Zhang et al. [32] gave a novel infinite-horizon optimal tracking control scheme for discrete-time nonlinear systems via greedy HDP algorithm. Dierks and Jagannathan [31] utilized the NDP technique to solve the HJB equation forward-in-time for optimal tracking control of affine nonlinear systems. However, to the best of our knowledge, there is still no result to solve the finite-horizon optimal tracking control problem for discrete-time nonlinear systems based on iterative ADP algorithm via HDP technique (iterative HDP algorithm for brief). In this paper, for the first time, we will provide an iterative ADP algorithm to design finite-horizon near-optimal tracking controller for a class of discrete-time nonlinear systems.

The rest of this paper is organized as follows. In Section 2, we present the problem statement, transform the finite-horizon optimal tracking control problem into an optimal regulation problem, and introduce the DTHJB equation for nonlinear systems. Section 3 starts by deriving the iterative ADP algorithm with convergence analysis, and then the finite-horizon optimal tracking control scheme is proposed which makes the cost function close to its optimal value within an ε-error bound. In Section 4, the NN implementation of the iterative ADP algorithm is presented. In Section 5, two examples are given to substantiate the theoretical results. Section 6 contains concluding remarks.

2. Problem statement

Consider the discrete-time nonlinear systems given by

$$x_{k+1}=f(x_k)+g(x_k)u_p(x_k), \tag{1}$$

where $x_k\in\mathbb{R}^n$ is the state, $u_p(x_k)\in\mathbb{R}^m$ is the control vector, $f(\cdot)$ and $g(\cdot)$ are differentiable in their argument with $f(0)=0$. Assume that $f+gu_p$ is Lipschitz continuous on a set Ω in $\mathbb{R}^n$ containing the origin, and that the system (1) is controllable in the sense that there exists a continuous control on Ω that asymptotically stabilizes the system. In the following part, $u_p(x_k)$ is denoted by u_{pk} for simplicity.

The objective for optimal tracking control problem is to determine optimal control law u_p^*, so as to make the nonlinear system (1) to track a reference (or desired) trajectory r_k in an optimal manner. Here, we assume that the reference trajectory r_k satisfies

$$r_{k+1}=\phi(r_k), \tag{2}$$

where $r_k\in\mathbb{R}^n$ and $\phi(r_k)\in\mathbb{R}^n$. Then, we define the tracking error as

$$e_k=x_k-r_k. \tag{3}$$

Inspired by the work of [31,32,36], we define the steady control corresponding to the reference trajectory r_k as

$$u_{dk}=g^{-1}(r_k)(\phi(r_k)-f(r_k)), \tag{4}$$

where $g^{-1}(r_k)g(r_k)=I_m$ and I_m is an $m\times m$ identity matrix.

By denoting

$$u_k=u_{pk}-u_{dk} \tag{5}$$

and using (1)–(4), we obtain

$$\begin{cases} e_{k+1}=f(e_k+r_k)+g(e_k+r_k)g^{-1}(r_k)(\phi(r_k),\\ \qquad -f(r_k))-\phi(r_k)+g(e_k+r_k)u_k\\ r_{k+1}=\phi(r_k) \end{cases} \tag{6}$$

as the new system. Note that in system (6), e_k and r_k are regarded as the system variables while u_k is seen as system input. The second equation of system (6) only gives the evolution of the reference trajectory which is not affected by the system input. Therefore, for simplicity, (6) can be rewritten as

$$e_{k+1}=F(e_k,u_k). \tag{7}$$

Now, let e_0 be an initial state of system (7) and define $\underline{u}_0^{N-1}=(u_0,u_1,\ldots,u_{N-1})$ be a control sequence with which the system (7) gives a trajectory starting from e_0: $e_1, e_2, \ldots, e_N$. We call the number of elements in the control sequence $\underline{u}_0^{N-1}$ the length of $\underline{u}_0^{N-1}$ and denote it as $|\underline{u}_0^{N-1}|$. Then, $|\underline{u}_0^{N-1}|=N$. The final state under the control sequence $\underline{u}_0^{N-1}$ is denoted as $e^{(f)}(e_0,\underline{u}_0^{N-1})=e_N$.

Definition 1. A nonlinear dynamical system is said to be stabilizable on a compact set $\Omega\in\mathbb{R}^n$, if for all initial conditions $e_0\in\Omega$, there exists a control sequence $\underline{u}_0^{N-1}=(u_0,u_1,\ldots,u_{N-1})$, $u_i\in\mathbb{R}^m$, $i=0,1,\ldots,N-1$, such that the state $e^{(f)}(e_0,\underline{u}_0^{N-1})=0$.

Let $\underline{u}_k^{N-1}=(u_k,u_{k+1},\ldots,u_{N-1})$ be the control sequence starting at k with length $N-k$. For finite-horizon optimal tracking control problem, it is desired to find the control sequence which minimizes the following cost function:

$$J(e_k,\underline{u}_k^{N-1})=\sum_{i=k}^{N-1}U(e_i,u_i), \tag{8}$$

where U is the utility function, $U(0,0)=0$, $U(e_i,u_i)\geq 0$ for $\forall e_i,u_i$. In this paper, the utility function is chosen as the quadratic form as follows:

$$U(e_i,u_i)=e_i^TQe_i+u_i^TRu_i.$$

This quadratic cost function can not only force the system state to follow the reference trajectory, but also force the system input to be close to the steady value in maintaining the state to its reference value. In fact, it can also be expressed as

$$U(e_i,u_i)=[e_i^T\ r_i^T]\begin{bmatrix}Q & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}e_i\\ r_i\end{bmatrix}+u_i^TRu_i,$$

when considered from the angle of system (6).

In this sense, the nonlinear tracking problem is converted into a regulation problem and the finite-horizon cost function for tracking is written in terms of e_k and u_k. Then, the problem of solving the finite-horizon optimal tracking control law u_p^* for system (1) is transformed into seeking the finite-horizon optimal control law u^* for system (7) with respect to (8). As a result, we will focus on how to design u^* in the following sections.

For finite-horizon optimal control problems, the designed feedback control must be finite-horizon admissible, which means it must not only stabilize the controlled system on Ω within finite number of time steps but also guarantee the cost function to be finite.

Definition 2. A control sequence $\underline{u}_k^{N-1}$ is said to be finite- horizon admissible for a state $e_k\in\mathbb{R}^n$ with respect to (8) on Ω if $\underline{u}_k^{N-1}$ is continuous on a compact set $\Omega_u\in\mathbb{R}^m$, $u(0)=0$, $e^{(f)}(e_k,\underline{u}_k^{N-1})=0$ and $J(e_k,\underline{u}_k^{N-1})$ is finite.

Let

$$\mathfrak{U}_{e_k}=\{\underline{u}_k: e^{(f)}(e_k,\underline{u}_k)=0\}$$

be the set of all finite-horizon admissible control sequences of e_k and

$$\mathfrak{U}_{e_k}^{(i)}=\{\underline{u}_k^{k+i-1}: e^{(f)}(e_k,\underline{u}_k^{k+i-1})=0, |\underline{u}_k^{k+i-1}|=i\}$$

be the set of all finite-horizon admissible control sequences of e_k with length i. Define the optimal cost function as

$$J^*(e_k)=\inf_{\underline{u}_k}\{J(e_k,\underline{u}_k): \underline{u}_k\in\mathfrak{U}_{e_k}\}. \tag{9}$$

Note that Eq. (8) can be written as

$$J(e_k,\underline{u}_k^{N-1}) = e_k^T Q e_k + u_k^T R u_k + \sum_{i=k+1}^{N-1} U(e_i,u_i) = e_k^T Q e_k + u_k^T R u_k + J(e_{k+1},\underline{u}_{k+1}^{N-1}). \quad (10)$$

Then, according to Bellman's optimality principle, it is known that the optimal cost function $J^*(e_k)$ satisfies the DTHJB equation

$$J^*(e_k) = \min_{u_k}\{e_k^T Q e_k + u_k^T R u_k + J^*(e_{k+1})\}. \quad (11)$$

The optimal control u^* satisfies the first-order necessary condition, which is given by the gradient of the right-hand side of (11) with respect to u_k. Then,

$$u^*(e_k) = -\frac{1}{2} R^{-1} g^T(e_k+r_k)\frac{\partial J^*(e_{k+1})}{\partial e_{k+1}}. \quad (12)$$

By substituting (12) into (11), the DTHJB equation becomes

$$J^*(e_k) = e_k^T Q e_k + \frac{1}{4}\left(\frac{\partial J^*(e_{k+1})}{\partial e_{k+1}}\right)^T g(e_k+r_k)R^{-1} \times g^T(e_k+r_k)\frac{\partial J^*(e_{k+1})}{\partial e_{k+1}} + J^*(e_{k+1}), \quad (13)$$

where $J^*(e_k)$ is the optimal cost function corresponding to the optimal control law $u^*(e_k)$. Since the above DTHJB equation cannot be solved exactly, we will present a novel algorithm to approximate the cost function iteratively in next section. Before that, we make the following assumption.

Assumption 1. For system (6), the inverse of the control coefficient matrix $g(e_k+r_k)$ exists.

Assumption 1 make sure that for given e_k and r_k, there exists an initial control u_k which can derive e_k to zero in one time step.

3. Finite-horizon neuro-optimal tracking control based on iterative ADP algorithm

Four subsections are included in this section. The iterative ADP algorithm is introduced in the first subsection, while in the second subsection, the corresponding convergence proof is developed. Then, the ε-optimal control algorithm and its design procedure are described in the third and fourth subsections, respectively.

3.1. Derivation of the iterative ADP algorithm

In this part, we present the iterative ADP algorithm, where the cost function and the control law are updated by recursive iterations.

First, we start with the initial cost function $V_0(\cdot)=0$, and then solve for the law of single control vector $v_0(e_k)$ as follows:

$$v_0(e_k) = \arg\min_{u_k}\{U(e_k,u_k)+V_0(e_{k+1})\} \quad \text{subject to } F(e_k,u_k)=0. \quad (14)$$

Once the control law $v_0(e_k)$ is determined, we update the cost function as

$$V_1(e_k) = \min_{u_k}\{U(e_k,u_k)+V_0(e_{k+1})\} = U(e_k,v_0(e_k)),$$

which can also be written as

$$V_1(e_k) = \min_{u_k} U(e_k,u_k) \text{ subject to } F(e_k,u_k)=0 = U(e_k,v_0(e_k)). \quad (15)$$

Then, for $i=1,2,\ldots$, the iterative algorithm can be implemented between the control law

$$v_i(e_k) = \arg\min_{u_k}\{U(e_k,u_k)+V_i(e_{k+1})\} = -\frac{1}{2}R^{-1}g^T(e_k+r_k)\frac{\partial V_i(e_{k+1})}{\partial e_{k+1}} \quad (16)$$

and the cost function

$$V_{i+1}(e_k) = \min_{u_k}\{U(e_k,u_k)+V_i(e_{k+1})\} = U(e_k,v_i(e_k))+V_i(F(e_k,v_i(e_k))). \quad (17)$$

Remark 1. In the iterative ADP algorithm (14)–(17), i is the iteration index of the control law and the cost function, while k is the time index of system's control and state trajectories. The cost function and control law are updated until they converge to the optimal ones.

Next, we will present a convergence proof of the iteration between (16) and (17) with the cost function $V_i \to J^*$ and the control law $v_i \to u^*$ as $i\to\infty$. Before that, we will see what $V_{i+1}(e_k)$ will be when it is expanded. According to (15) and (17), we can obtain

$$\begin{aligned} V_{i+1}(e_k) &= \min_{u_k}\{U(e_k,u_k)+V_i(e_{k+1})\} \\ &= \min_{\underline{u}_k^{k+1}}\{U(e_k,u_k)+U(e_{k+1},u_{k+1})+V_{i-1}(e_{k+2})\} \\ &\vdots \\ &= \min_{\underline{u}_k^{k+i-1}}\{U(e_k,u_k)+U(e_{k+1},u_{k+1}) \\ &\quad +\cdots+U(e_{k+i-1},u_{k+i-1})+V_1(e_{k+i})\}, \end{aligned} \quad (18)$$

where

$$V_1(e_{k+i}) = \min_{u_{k+i}} U(e_{k+i},u_{k+i}) \quad \text{subject to } F(e_{k+i},u_{k+i})=0. \quad (19)$$

Then, we have

$$\begin{aligned} V_{i+1}(e_k) &= \min_{\underline{u}_k^{k+i}}\sum_{j=0}^{i} U(e_{k+j},u_{k+j}) \\ &\text{subject to } F(e_{k+i},u_{k+i})=0 \\ &= \min_{\underline{u}_k^{k+i}}\left\{J(e_k,\underline{u}_k^{k+i}) : \underline{u}_k^{k+i}\in\mathfrak{U}_{e_k}^{(i+1)}\right\}, \end{aligned} \quad (20)$$

which can also be written as

$$V_{i+1}(e_k) = \sum_{j=0}^{i} U(e_{k+j},v_{i-j}(e_{k+j})) \quad (21)$$

when using the notation in (16). These equations will be useful in the convergence proof of the iterative ADP algorithm.

3.2. Convergence analysis of the iterative ADP algorithm

Theorem 1. *Suppose $\mathfrak{U}_{e_k}^{(1)}\neq\emptyset$. Then, the cost function sequence $\{V_i\}$ obtained by (14)–(17) is a monotonically nonincreasing sequence satisfying $V_{i+1}(e_k)\le V_i(e_k)$ for $\forall i\ge 1$, i.e., $V_1(e_k)=\max\{V_i(e_k): i=1,2,\ldots\}$.*

Proof. We prove this theorem by mathematical induction. First, we let $i=1$. The cost function $V_1(e_k)$ is given in (15) and the finite-horizon admissible control sequence is $\hat{\underline{u}}_k^k = (v_0(e_k))$. Now, we show that there exists a finite-horizon admissible control sequence $\hat{\underline{u}}_k^{k+1}$ with length 2 such that $J(e_k,\hat{\underline{u}}_k^{k+1}) = V_1(e_k)$. Let $\hat{\underline{u}}_k^{k+1} = (\hat{\underline{u}}_k^k, 0)$, then $|\hat{\underline{u}}_k^{k+1}| = 2$. Since $e_{k+1} = F(e_k,v_0(e_k)) = 0$ and $\hat{u}_{k+1}=0$, we have $e_{k+2} = F(e_{k+1},\hat{u}_{k+1}) = F(0,0) = 0$. Thus, $\hat{\underline{u}}_k^{k+1}$ is a finite-horizon admissible control sequence. Since $U(e_{k+1},\hat{u}_{k+1}) = U(0,0) = 0$,

we can obtain

$$J(e_k,\underline{\hat{u}}_k^{k+1}) = U(e_k,v_0(e_k)) + U(e_{k+1},\hat{u}_{k+1}) = U(e_k,v_0(e_k)) = V_1(e_k).$$

On the other hand, according to (20), we have

$$V_2(e_k) = \min_{\underline{u}_k^{k+1}}\{J(e_k,\underline{u}_k^{k+1}) : \underline{u}_k^{k+1} \in \mathfrak{U}_{e_k}^{(2)}\},$$

which reveals that

$$V_2(e_k) \le J(e_k,\underline{\hat{u}}_k^{k+1}) = V_1(e_k). \tag{22}$$

Therefore, the theorem holds for $i=1$.

Next, assume that the theorem holds for any $i=q$, where $q>1$. The current cost function can be expressed as

$$V_q(e_k) = \sum_{j=0}^{q-1} U(e_{k+j},v_{q-1-j}(e_{k+j})),$$

where $\underline{\hat{u}}_k^{k+q-1} = (v_{q-1}(e_k),v_{q-2}(e_{k+1}),\ldots,v_0(e_{k+q-1}))$ is the corresponding finite-horizon admissible control sequence.

Then, for $i=q+1$, we can construct a control sequence $\underline{\hat{u}}_k^{k+q} = (v_{q-1}(e_k),v_{q-2}(e_{k+1}),\ldots,v_0(e_{k+q-1}),0)$ with length $q+1$, under which the error trajectory is given as e_k, $e_{k+1} = F(e_k, v_{q-1}(e_k))$, $e_{k+2} = F(e_{k+1},v_{q-2}(e_{k+1})),\ldots,$ $e_{k+q} = F(e_{k+q-1}, v_0(e_{k+q-1})) = 0$, $e_{k+q+1} = F(e_{k+q},\hat{u}_{k+q}) = F(0,0) = 0$. This shows that $\underline{\hat{u}}_k^{k+q}$ is a finite-horizon admissible control sequence. As $U(e_{k+q},\hat{u}_{k+q}) = U(0,0) = 0$, we can acquire

$$\begin{aligned} J(e_k,\underline{\hat{u}}_k^{k+q}) &= U(e_k,v_{q-1}(e_k)) + U(e_{k+1},v_{q-2}(e_{k+1})) \\ &\quad + \cdots + U(e_{k+q-1},v_0(e_{k+q-1})) + U(e_{k+q},\hat{u}_{k+q}) \\ &= \sum_{j=0}^{q-1} U(e_{k+j},v_{q-1-j}(e_{k+j})) = V_q(e_k). \end{aligned}$$

On the other hand, according to (20), we have

$$V_{q+1}(e_k) = \min_{\underline{u}_k^{k+q}}\{J(e_k,\underline{u}_k^{k+q}) : \underline{u}_k^{k+q} \in \mathfrak{U}_{e_k}^{(q+1)}\},$$

which implies that

$$V_{q+1}(e_k) \le J(e_k,\underline{\hat{u}}_k^{k+q}) = V_q(e_k). \tag{23}$$

Accordingly, we complete the proof by mathematical induction. □

We have concluded that the cost function sequence $\{V_i(e_k)\}$ is a monotonically nonincreasing sequence which is bounded below, and therefore, its limit exists. Here, we denote it as $V_\infty(e_k)$, i.e., $\lim_{i\to\infty} V_i(e_k) = V_\infty(e_k)$. Next, let us consider what will happen when we make $i\to\infty$ in (17).

Theorem 2. *For any discrete time step k and tracking error e_k, the following equation holds*:

$$V_\infty(e_k) = \min_{u_k}\{U(e_k,u_k) + V_\infty(e_{k+1})\}. \tag{24}$$

Proof. For any admissible control $\tau_k = \tau(e_k)$ and i, according to Theorem 1 and (17), we have

$$V_\infty(e_k) \le V_{i+1}(e_k) = \min_{u_k}\{U(e_k,u_k) + V_i(e_{k+1})\} \le U(e_k,\tau_k) + V_i(e_{k+1}).$$

Let $i\to\infty$, we get

$$V_\infty(e_k) \le U(e_k,\tau_k) + V_\infty(e_{k+1}).$$

Note that in the above equation, τ_k is chosen arbitrarily. Thus, we can obtain

$$V_\infty(e_k) \le \min_{u_k}\{U(e_k,u_k) + V_\infty(e_{k+1})\}. \tag{25}$$

On the other hand, let $\delta>0$ be an arbitrary positive number. Then, there exists a positive integer l such that

$$V_l(e_k) - \delta \le V_\infty(e_k) \le V_l(e_k) \tag{26}$$

because $V_i(e_k)$ is nonincreasing for $i\ge 1$ with $V_\infty(e_k)$ as its limit. Besides, from (17), we can acquire

$$\begin{aligned} V_l(e_k) &= \min_{u_k}\{U(e_k,u_k) + V_{l-1}(e_{k+1})\} \\ &= U(e_k,v_{l-1}(e_k)) + V_{l-1}(F(e_k,v_{l-1}(e_k))). \end{aligned}$$

Combining with (26), we can obtain

$$\begin{aligned} V_\infty(e_k) &\ge U(e_k,v_{l-1}(e_k)) + V_{l-1}(F(e_k,v_{l-1}(e_k))) - \delta \\ &\ge U(e_k,v_{l-1}(e_k)) + V_\infty(F(e_k,v_{l-1}(e_k))) - \delta \\ &\ge \min_{u_k}\{U(e_k,u_k) + V_\infty(e_{k+1})\} - \delta, \end{aligned}$$

which reveals that

$$V_\infty(e_k) \ge \min_{u_k}\{U(e_k,u_k) + V_\infty(e_{k+1})\} \tag{27}$$

because of the arbitrariness of δ. Based on (25) and (27), we can conclude that (24) is true. □

Next, we will prove that the cost function sequence $\{V_i\}$ converges to the optimal cost function J^* as $i\to\infty$.

Theorem 3. *Define the cost function sequence $\{V_i\}$ as in (17) with $V_0(\cdot)=0$. If the system state e_k is controllable, then J^* is the limit of the cost function sequence $\{V_i\}$, i.e.,*

$$V_\infty(e_k) = J^*(e_k).$$

Proof. On one hand, in accordance with (9) and (20), we can acquire

$$\begin{aligned} J^*(e_k) &= \inf_{\underline{u}_k}\{J(e_k,\underline{u}_k) : \underline{u}_k \in \mathfrak{U}_{e_k}\} \\ &\le \min_{\underline{u}_k^{k+i-1}}\{J(e_k,\underline{u}_k^{k+i-1}) : \underline{u}_k^{k+i-1} \in \mathfrak{U}_{e_k}^{(i)}\} = V_i(e_k). \end{aligned}$$

Letting $i\to\infty$, we get

$$J^*(e_k) \le V_\infty(e_k). \tag{28}$$

On the other hand, according to the definition of $J^*(e_k)$, for any $\eta>0$, there exists an admissible control sequence $\underline{\sigma}_k \in \mathfrak{U}_{e_k}$ such that

$$J(e_k,\underline{\sigma}_k) \le J^*(e_k) + \eta. \tag{29}$$

Now, we suppose that $|\underline{\sigma}_k| = q$, which shows that $\underline{\sigma}_k \in \mathfrak{U}_{e_k}^{(q)}$. Then, we can obtain

$$\begin{aligned} V_\infty(e_k) \le V_q(e_k) &= \min_{\underline{u}_k^{k+q-1}}\{J(e_k,\underline{u}_k^{k+q-1}) : \underline{u}_k^{k+q-1} \in \mathfrak{U}_{e_k}^{(q)}\} \\ &\le J(e_k,\underline{\sigma}_k), \end{aligned}$$

using Theorem 1 and (20). Combining with (29), we get

$$V_\infty(e_k) \le J^*(e_k) + \eta.$$

Noticing that η is chosen arbitrarily in the above expression, we have

$$V_\infty(e_k) \le J^*(e_k). \tag{30}$$

Based on (28) and (30), we can conclude that $J^*(e_k)$ is the limit of the cost function sequence $\{V_i\}$ as $i\to\infty$, i.e., $V_\infty(e_k) = J^*(e_k)$. □

From Theorems 1–3, we can obtain that the cost function sequence $\{V_i(e_k)\}$ converges to the optimal cost function $J^*(e_k)$ of the DTHJB equation, i.e., $V_i\to J^*$ as $i\to\infty$. Then, according to (12) and (16), we can conclude the convergence of the corresponding control law sequence. Now, we present the following corollary.

Corollary 1. *Define the cost function sequence $\{V_i\}$ as in (17) with $V_0(\cdot)=0$, and the control law sequence $\{v_i\}$ as in (16). If the system state e_k is controllable, then the sequence $\{v_i\}$ converges to the*

optimal control law u^* as $i\to\infty$, i.e.,

$$\lim_{i\to\infty} v_i(e_k) = u^*(e_k).$$

3.3. The ε-optimal control algorithm

According to Theorems 1–3 and Corollary 1, we should run the iterative ADP algorithm (14)–(17) until $i\to\infty$ to obtain the optimal cost function $J^*(e_k)$, and then to get a control vector $v_\infty(e_k)$ based on which we can construct a control sequence $\underline{u}_\infty(e_k)=(v_\infty(e_k),v_\infty(e_{k+1}),\ldots,v_\infty(e_{k+i}),\ldots)$ to control the state to reach the target. Obviously, $\underline{u}_\infty(e_k)$ has infinite length. Though it is feasible in terms of theory, it is always not practical to do so because most real world systems need to be effectively controlled within finite-horizon. Therefore, in this section, we will propose a novel ε-optimal control strategy using the iterative ADP algorithm to deal with the problem. The idea is, for a given error bound $\varepsilon>0$, the iterative number i will be chosen so that the error between $V_i(e_k)$ and $J^*(e_k)$ is within the bound.

Let $\varepsilon>0$ be any small number, e_k be any controllable state, and $J^*(e_k)$ be the optimal value of the cost function sequence defined as in (17). From Theorem 3, it is clear that there exists a finite i such that

$$|V_i(e_k)-J^*(e_k)|\le\varepsilon. \tag{31}$$

The length of the optimal control sequence starting from e_k with respect to ε is defined as

$$K_\varepsilon(e_k)=\min\{i:|V_i(e_k)-J^*(e_k)|\le\varepsilon\}. \tag{32}$$

The corresponding control law

$$\begin{aligned} v_{i-1}(e_k) &= \arg\min_{u_k}\{U(e_k,u_k)+V_{i-1}(e_{k+1})\}\\ &= -\frac{1}{2}R^{-1}g^T(e_k+r_k)\frac{\partial V_{i-1}(e_{k+1})}{\partial e_{k+1}} \end{aligned} \tag{33}$$

is called the ε-optimal control and is denoted as $\mu_\varepsilon^*(e_k)$.

In this sense, we can see that an error ε between $V_i(e_k)$ and $J^*(e_k)$ is introduced into the iterative ADP algorithm, which makes the cost function sequence $\{V_i(e_k)\}$ converge in finite number of iteration steps.

However, the optimal criterion (31) is difficult to verify because the optimal cost function $J^*(e_k)$ is unknown in general. Consequently, we will use an equivalent criterion, i.e.,

$$|V_i(e_k)-V_{i+1}(e_k)|\le\varepsilon \tag{34}$$

to replace (31).

In fact, if $|V_i(e_k)-J^*(e_k)|\le\varepsilon$ holds, we have $V_i(e_k)\le J^*(e_k)+\varepsilon$. Combining with $J^*(e_k)\le V_{i+1}(e_k)\le V_i(e_k)$, we can find that

$$0\le V_i(e_k)-V_{i+1}(e_k)\le\varepsilon,$$

which means

$$|V_i(e_k)-V_{i+1}(e_k)|\le\varepsilon.$$

On the other hand, according to Theorem 3, $|V_i(e_k)-V_{i+1}(e_k)|\to0$ connotes that $V_i(e_k)\to J^*(e_k)$. As a result, if $|V_i(e_k)-V_{i+1}(e_k)|\le\varepsilon$ holds for any given small ε, we can derive the conclusion that $|V_i(e_k)-J^*(e_k)|\le\varepsilon$ holds if i is sufficiently large.

3.4. Design procedure of the finite-horizon optimal tracking control scheme using iterative ADP algorithm

In this section, we will give the detailed design procedure for the finite-horizon nonlinear optimal tracking control scheme using the iterative ADP algorithm.

Step 1 Specify an error bound ε for the given initial state x_0. Choose $i_{\max}$, the reference trajectory r_k, and the matrices Q and R.
Step 2 Compute e_k according to (2) and (3).
Step 3 Set $i=0$, $V_0(e_k)=0$. Obtain the initial finite-horizon admissible vector $v_0(e_k)$ by (14) and update the cost function $V_1(e_k)$ by (15).
Step 4 Set $i=i+1$.
Step 5 Compute $v_i(e_k)$ by (16) and the corresponding cost function $V_{i+1}(e_k)$ by (17).
Step 6 If $|V_i(e_k)-V_{i+1}(e_k)|\le\varepsilon$, then go to Step 8; otherwise, go to Step 7.
Step 7 If $i>i_{\max}$, then go to Step 8; otherwise, go to Step 4.
Step 8 Stop.

After the optimal control law $u^*(e_k)$ for system (6) is derived under the given error bound ε, we can compute the optimal tracking control input for original system (1) by

$$u_{pk}^*=u^*(e_k)+u_{dk}=u^*(e_k)+g^{-1}(r_k)(\phi(r_k)-f(r_k)). \tag{35}$$

In the following section, we will describe the implementation of the iterative ADP algorithm based on NNs in detail.

4. NN implementation of the iterative ADP algorithm via HDP technique

Now, we implement the iterative HDP algorithm in (14)–(17) using NNs. In the iterative HDP algorithm, there are three networks, which are model network, critic network and action network. All the networks are chosen as three-layer feedforward NNs. The inputs of the critic network and the action network are e_k, while the inputs of the model network are e_k and $\hat{v}_i(e_k)$. The structure diagram of the iterative HDP algorithm is shown in Fig. 1.

4.1. The model network

The purpose of designing the model network is to approximate the error d ynamics. We should train the model network before carrying out the iterative HDP algorithm. For given e_k and $\hat{v}_i(e_k)$, we can obtain the output of the model network as

$$\hat{e}_{k+1}=\omega_m^T\sigma(v_m^Tz_k), \tag{36}$$

where

$$z_k=[e_k^T\ \hat{v}_i^T(e_k)]^T.$$

We define the error function of the model network as

$$e_{mk}=\hat{e}_{k+1}-e_{k+1}. \tag{37}$$

The weights in the model network are updated to minimize the following performance measure:

$$E_{mk}=\tfrac{1}{2}e_{mk}^Te_{mk}. \tag{38}$$

Using the gradient-based adaptation rule, the weights can be updated as

$$\omega_m(j+1)=\omega_m(j)-\alpha_m\left[\frac{\partial E_{mk}}{\partial\omega_m(j)}\right], \tag{39}$$

$$v_m(j+1)=v_m(j)-\alpha_m\left[\frac{\partial E_{mk}}{\partial v_m(j)}\right], \tag{40}$$

where $\alpha_m>0$ is the learning rate of the model network, and j is the iterative step for updating the weight parameters.

After the model network is trained, its weights are kept unchanged.

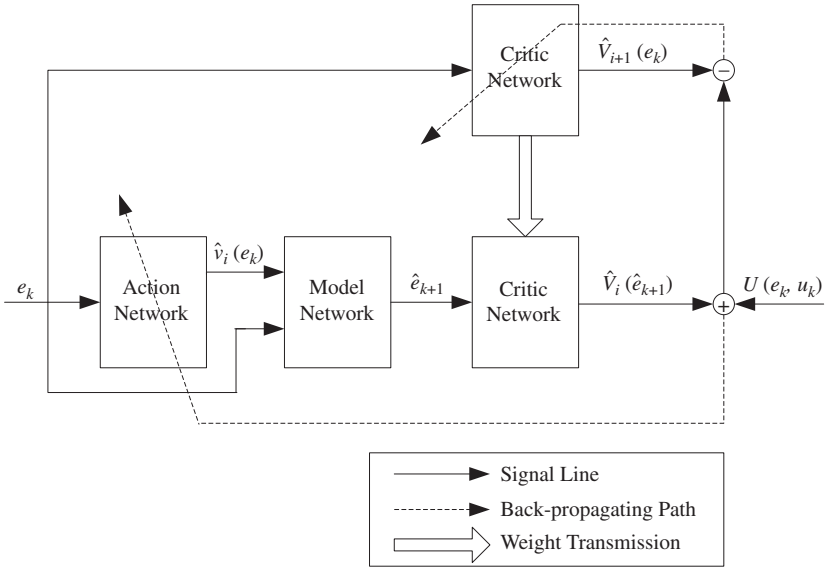

Fig. 1. The structure diagram of the iterative HDP algorithm.

4.2. The critic network

The critic network is used to approximate the cost function $V_i(e_k)$. The output of the critic network is denoted as

$$\hat{V}_i(e_k) = \omega_{ci}^T \sigma(v_{ci}^T e_k). \tag{41}$$

The target function can be written as

$$V_i(e_k) = e_k^T Q e_k + v_{i-1}^T(e_k) R v_{i-1}(e_k) + \hat{V}_{i-1}(\hat{e}_{k+1}). \tag{42}$$

Then, we define the error function for the critic network as

$$e_{cik} = \hat{V}_i(e_k) - V_i(e_k). \tag{43}$$

The objective function to be minimized for the critic network is

$$E_{cik} = \tfrac{1}{2} e_{cik}^T e_{cik}. \tag{44}$$

The weight updating rule for training the critic network is also gradient-based adaptation given by

$$\omega_{ci}(j+1) = \omega_{ci}(j) - \alpha_c \left[\frac{\partial E_{cik}}{\partial \omega_{ci}(j)}\right], \tag{45}$$

$$v_{ci}(j+1) = v_{ci}(j) - \alpha_c \left[\frac{\partial E_{cik}}{\partial v_{ci}(j)}\right], \tag{46}$$

where $\alpha_c > 0$ is the learning rate of the critic network, and j is the inner-loop iterative step for updating the weight parameters.

4.3. The action network

In the action network, the state e_k is used as input to obtain the optimal control as the output of the network. The output can be formulated as

$$\hat{v}_i(e_k) = \omega_{ai}^T \sigma(v_{ai}^T e_k). \tag{47}$$

The target control input is given by

$$v_i(e_k) = -\frac{1}{2} R^{-1} g^T(e_k + r_k) \frac{\partial \hat{V}_i(\hat{e}_{k+1})}{\partial \hat{e}_{k+1}}. \tag{48}$$

The error function of the action network can be defined as

$$e_{aik} = \hat{v}_i(e_k) - v_i(e_k). \tag{49}$$

The weights of the action network are updated to minimize the following performance error measure:

$$E_{aik} = \tfrac{1}{2} e_{aik}^T e_{aik}. \tag{50}$$

Similarly, the weight updating algorithm is

$$\omega_{ai}(j+1) = \omega_{ai}(j) - \alpha_a \left[\frac{\partial E_{aik}}{\partial \omega_{ai}(j)}\right], \tag{51}$$

$$v_{ai}(j+1) = v_{ai}(j) - \alpha_a \left[\frac{\partial E_{aik}}{\partial v_{ai}(j)}\right], \tag{52}$$

where $\alpha_a > 0$ is the learning rate of the action network, and j is the inner-loop iterative step for updating the weight parameters.

5. Simulation study

In this section, two simulation examples are provided to confirm the theoretical results.

5.1. Example 1

The first example is derived from [31] with some modifications. Consider the following nonlinear system:

$$x_{k+1} = f(x_k) + g(x_k) u_{pk}, \tag{53}$$

where $x_k = [x_{1k}\ x_{2k}]^T \in \mathbb{R}^2$ and $u_{pk} = [u_{p1k}\ u_{p2k}]^T \in \mathbb{R}^2$ are the state and control variables, respectively. The parameters of the cost function are chosen as $Q = 0.5I$ and $R = 2I$, where I denotes the identity matrix with suitable dimensions. The state of the controlled system is initialized to be $x_0 = [0.8\ -0.5]^T$. The system functions are given as

$$f(x_k) = \begin{bmatrix} \sin(0.5x_{2k}) x_{1k}^2 \\ \cos(1.4x_{2k}) \sin(0.9x_{1k}) \end{bmatrix},$$

$$g(x_k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The reference trajectory for the above system is selected as

$$r_k = \begin{bmatrix} \sin(0.25k) \\ \cos(0.25k) \end{bmatrix}.$$

We set the error bound of the iterative HDP algorithm as $\varepsilon = 10^{-5}$ and implement the algorithm at time instant $k = 0$. The initial control vector of system (6) can be computed as $v_0(e_0) = [0.64\ \sin(0.25) - \sin(0.72)\cos(0.7)]^T$, where $e_0 = [0.8\ -\ 0.5]^T$. Then, we choose three-layer feedforward NNs as model network, critic

network and action network with the structures 4–8–2, 2–8–1, 2–8–2, respectively. The initial weights of the three networks are all set to be random in $[-1,1]$. It should be mentioned that the model network should be trained first. We train the model network for 1000 steps using 500 data samples under the learning rate $\alpha_m = 0.1$. After the training of the model network is completed, the weights are kept unchanged. Then, we train the critic network and the action network for 20 iterations (i.e., for $i = 1,2,\ldots,20$) with each iteration of 2000 training steps to make sure the given error bound $\varepsilon = 10^{-5}$ is reached. In the training process, the learning rate $\alpha_c = \alpha_a = 0.05$. The convergence process of the cost function of the iterative HDP algorithm is shown in Fig. 2, for $k=0$. We can see that the iterative cost function sequence does converge to the optimal cost function quite rapidly, which indicates the effectiveness of the iterative HDP algorithm. Therefore, we have $|V_{19}(e_0)-V_{20}(e_0)| \le \varepsilon$, which means that the number of steps of the ε-optimal control is $K_\varepsilon(e_0) = 19$. Besides, the ε-optimal control law $\mu_\varepsilon^*(e_0)$ for system (6) can also be obtained during the iteration process.

Next, we compute the near-optimal tracking control law for original system (1) using (35) and apply it to the controlled system for 40 time steps. The obtained state curves are shown in

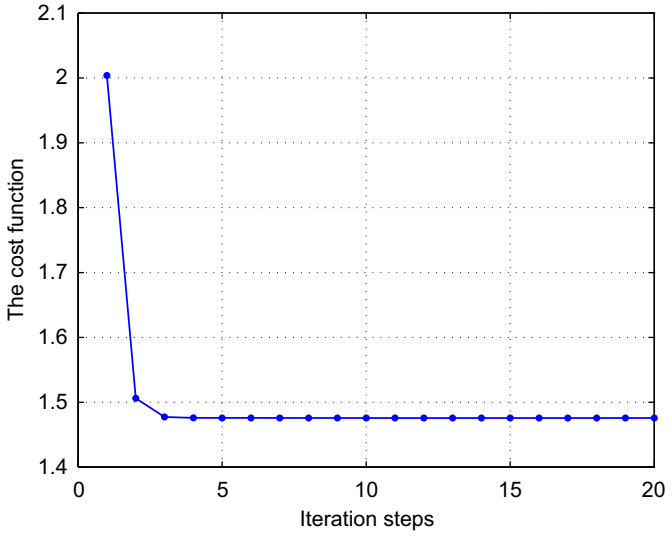

Fig. 2. The convergence process of the cost function.

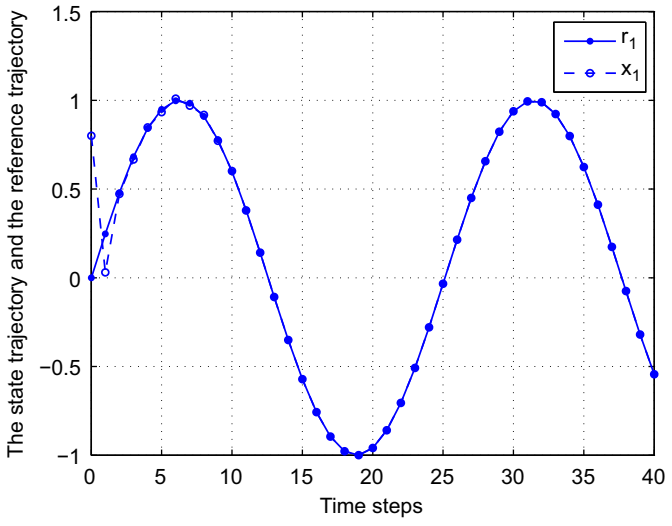

Fig. 3. The state trajectory x_1 and the reference trajectory r_1.

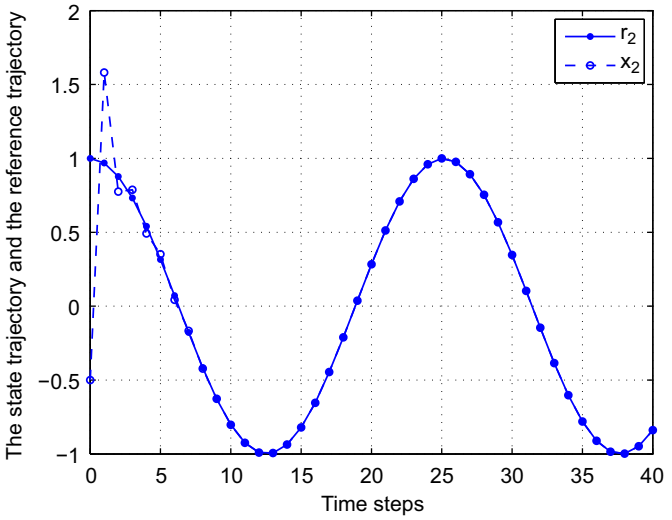

Fig. 4. The state trajectory x_2 and the reference trajectory r_2.

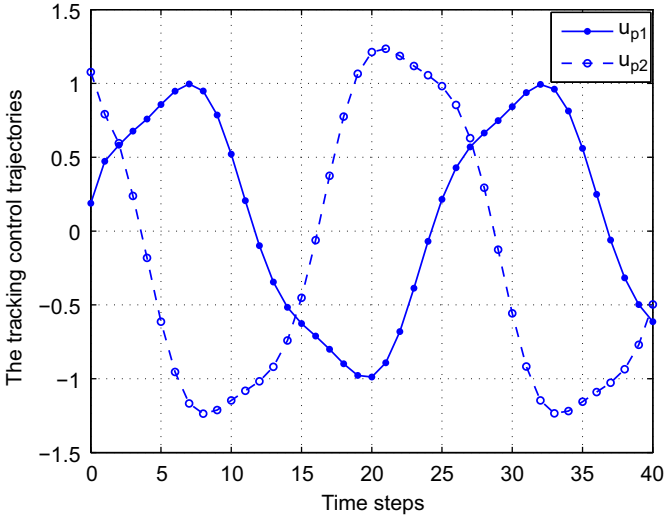

Fig. 5. The tracking control trajectories u_p.

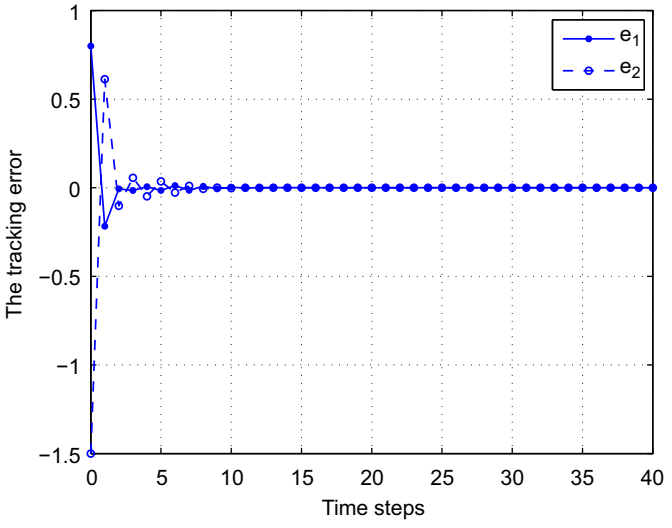

Fig. 6. The tracking error e.

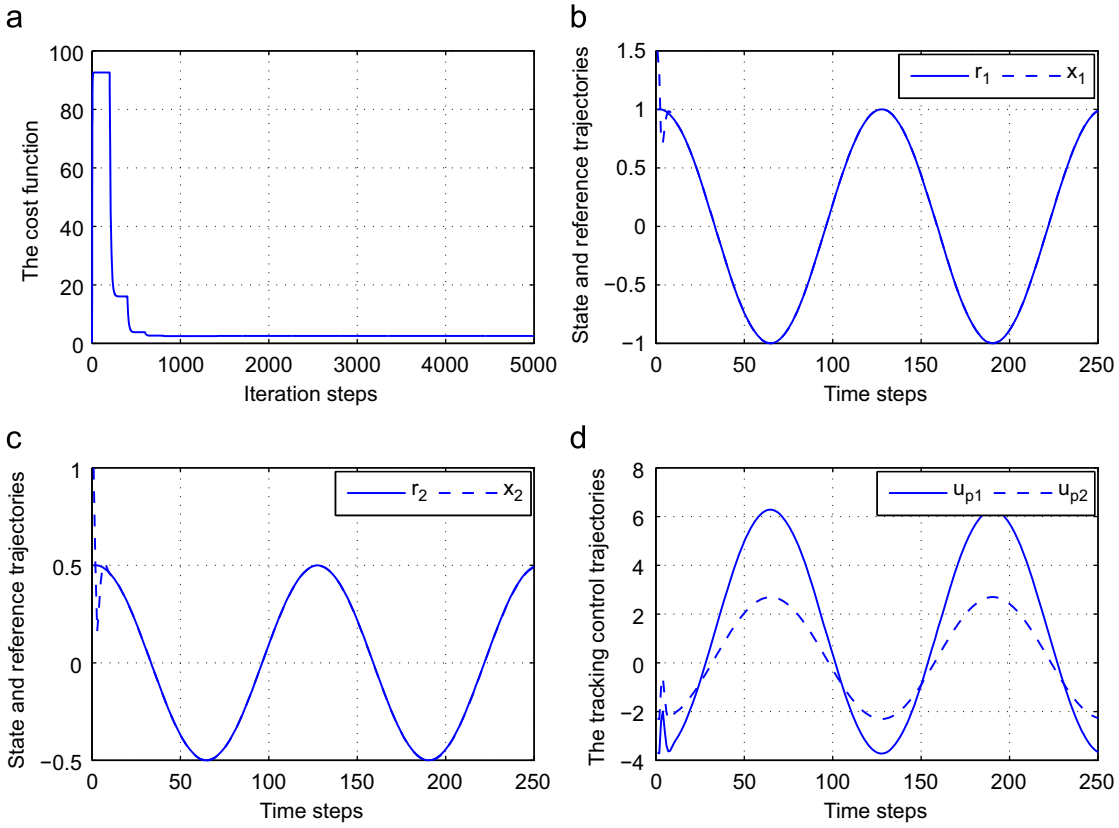

Fig. 7. Simulation results of Example 2.

Figs. 3 and 4, where the corresponding reference trajectories are also plotted to evaluate the tracking performance. The tracking control curves and the tracking errors are shown in Figs. 5 and 6, respectively. Besides, we can derive that the tracking error becomes $e_{19}=[0.2778\times10^{-5}-0.8793\times10^{-5}]^T$ after 19 time steps. These simulation results verify the excellent performance of the tracking controller developed by the iterative ADP algorithm.

5.2. Example 2

The second example is obtained from [32]. Consider the nonlinear discrete-time system described as (53) where

$$f(x_k)=\begin{bmatrix}0.2x_{1k}e^{x_{2k}^2}\\0.3x_{2k}^3\end{bmatrix},$$

$$g(x_k)=\begin{bmatrix}-0.2 & 0\\0 & -0.2\end{bmatrix}.$$

The desired trajectory is set to

$$r_k=\begin{bmatrix}\sin(k+0.5\pi)\\0.5\cos k\end{bmatrix}.$$

In the implementation of the iterative HDP algorithm, the initial weights and structures of three networks are set the same as Example 1. Then, for the given initial state $x_0=[1.5\ 1]^T$, we train the model network for 10 000 steps using 1000 data samples under the learning rate $\alpha_m=0.05$. Besides, the critic network and the action network are trained for 5000 iterations so that the given error bound $\varepsilon=10^{-6}$ is reached. The learning rate in the training process is also $\alpha_c=\alpha_a=0.05$.

The convergence process of the cost function of the iterative HDP algorithm is shown in Fig. 7(a), for $k=0$. Then, we apply the tracking control law to the system for 250 time steps and obtain the state and reference trajectories shown in Fig. 7(b) and (c). Besides, the tracking control curves are given in Fig. 7(d). It is clear from the simulation results that the iterative HDP algorithm proposed in this paper is very effective in solving the finite-horizon tracking control problems.

6. Conclusion

An effective method is proposed in this paper to design the finite-horizon near-optimal tracking controller for a class of discrete-time nonlinear systems. The iterative ADP algorithm is introduced to solve the cost function of the DTHJB equation with convergence analysis, which obtains a finite-horizon near-optimal tracking controller that makes the cost function close to its optimal value within an ε-error bound. Three NNs are used to approximate the cost function, the control law, and the nonlinear system, respectively. The simulation examples confirmed the validity of the tracking control approach. The strategy presented in this paper only be true of a class of affine nonlinear systems and requires complete knowledge of the system dynamics. Though there are many practical systems for which the approach can be applied, it is necessary to broaden its applicability for a more general class of nonlinear systems. Consequently, our future work includes studying the optimal tracking control problems for non-affine nonlinear systems and model-free systems.

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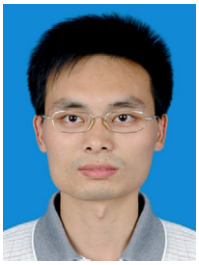

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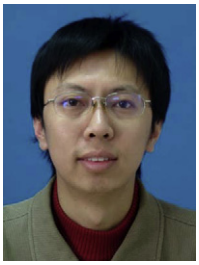

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