Multiuser Detection Using the Taguchi Method for DS-CDMA Systems

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***Abstract*—We study multiuser detection for direct-sequence code-division multiple-access systems in a multipath environment. Systems with unknown channel information are considered and the well-known result for maximum likelihood multiuser detector is directly used in our work. Due to the high computational cost of the maximum likelihood detector, most existing works have investigated simplified, linearized, and/or suboptimal solutions that have less computational requirements. In our approach, we use the Taguchi method that involves the use of orthogonal arrays in estimating the gradient of the likelihood function. The Taguchi method has been widely used in experimental designs for problems with multiple parameters where the optimization of a cost function is required. In this work, we choose the likelihood function as the cost function in the Taguchi method. The use of the Taguchi method for multiuser detection is a novel idea, and it leads to efficient algorithms that can find a satisfactory solution by maximizing the likelihood function in a small number of iterations. One of the advantages of the present Taguchi method is that it is blind since no channel estimation is required to detect the transmitted data, which is not the case in many existing methods. Simulation results show that the Taguchi multiuser detector significantly outperforms the conventional receivers, is insensitive to initial values of parameters, and has performance close to that of minimum mean square error detectors and decorrelating detectors. In addition, our algorithm is suitable for parallel implementations.**

***Index Terms*—Direct-sequence code-division multiple-access (DS-CDMA), maximum likelihood, multiuser detection, orthogonal array, Taguchi method.**

I. Introduction

THE Taguchi method of experimental design has been widely used in industry for the purpose of finding factors that are most important in achieving useful goals in a manufacturing process [4], [7], [14], [25]. Several factors that are related to the goals and are under the user's control are selected. These factors are varied over two or more levels in a systematic manner. Experiments are then designed according to an orthogonal array to show the effects of each potential primary factor. The Taguchi method involves an analysis that reveals which of the factors are most effective in reaching the goals and the directions in which these factors should be adjusted to improve the results. The control over achieving the goals will be best obtained by changes in these primary factors in the direction indicated by the analysis. The present paper applies the Taguchi method to the maximization of the likelihood function for multiuser detection in wireless communications.

Wireless communications for mobile telephone and data transmission are currently undergoing very rapid development. Many of the emerging wireless systems will incorporate considerable signal processing intelligence in order to provide advanced services such as multimedia transmission. As a result, a technique named direct-sequence code-division multiple-access (DS-CDMA) has become very popular in recent years. In DS-CDMA communication systems, different users employ distinct spreading codes and transmit at the same time and frequency. Therefore, multiple access interference (MAI) exists in the received signal, reducing the performance and creating "near-far" effects [23]. As the number of users increases, the MAI becomes substantial and the system capacity as well as the performance become interference limited.

Multiuser detectors [30] exploit the underlying structure induced by the spreading waveforms of the DS-CDMA user signals for interference suppression. Various linear and nonlinear multiuser detectors have been developed over the past decade. It has been well established that the optimal multiuser detector can substantially enhance the receiver performance and increase the capacity of CDMA systems. Optimum maximum likelihood multiuser detectors perform joint maximum likelihood estimation of the symbols transmitted by different users and are implemented using the Viterbi algorithm for single path channels in [18] and [29] and for multipath channels in [19]. In addition, optimum noncoherent maximum likelihood detectors for generalized multiuser diversity communications are studied in [26]. However, the implementation of optimum maximum likelihood multiuser detectors is prohibitively complex when the number of users is large. Thus, many suboptimal maximum likelihood detectors with lower computational complexity have been explored. A successive multistage joint detector is proposed in [6] for performing suboptimal maximum likelihood multiuser detection. Other suboptimal maximum likelihood multiuser detectors include: 1) the decorrelating detectors [10], [12], [13], [15] in which nulls in the space–time domain are directed at the interfering users; 2) an M-estimator-based multiuser detector [21] for flat-fading CDMA channels with impulsive non-Gaussian noise; 3) blind group multiuser detectors [24] using interference identification; 4) Hopfield neural-network-based multiuser detector [11]; and 5) genetic-algorithm-based algorithm [37] for estimating the transmitted

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symbols and fading channel coefficients based on the maximum likelihood principle. Beside the detectors based on maximum likelihood estimation, there are many nonmaximum likelihood multiuser detectors as well. The minimum mean square error (MMSE) receiver [8], [9], [16], [17], [39] is a popular linear nonmaximum likelihood detector, which minimizes the mean square error between the filter output and the transmitted bit and also maximizes the output signal-to-interference ratio. A blind adaptive multiuser detector based on constant modulus algorithm is presented in [20], and a blind adaptive detector based on minimum output energy is proposed in [27] and [36]. Both of these two blind detectors are shown to approximate the MMSE solutions. It is shown in [33] that based on signal subspace estimation, both decorrelating detectors and linear MMSE detectors can be obtained blindly. The decision feedback detector [28] is a nonlinear nonmaximum likelihood detector that cancels the in terference from the users that have already been decoded and suppresses interference from the users that have not yet been decoded. Nonlinear turbo soft interference cancellation multiuser detectors are developed in [5] and [34], which make use of both soft interference cancellation and instantaneous linear MMSE filtering. An iterative nonlinear detector based on multistage soft interference cancelers is proposed in [38]. The energy learning technique, called support vector machines is proposed in [1] as a method for obtaining a nonlinear multiuser detector from a relatively small training data block.

The goal of this paper is to develop for multiuser CDMA wireless systems receivers with reduced complexity by employing the Taguchi method based on orthogonal arrays. One of the advantages of the present Taguchi method is that it is blind since no channel estimation is required to detect the transmitted data, which is not the case in many existing methods. We recursively maximize the likelihood function [30] to recover the transmitted information. It turns out that the maximization of the likelihood function in [30] is an NP-hard optimization problem. However, in our approach, the objective function is optimized using the Taguchi method, in which only very limited computations are needed, assuming the knowledge of the desired users' spreading codes and delays. The present approach can be classified as a nonlinear suboptimal maximum likelihood method and it is of practical interest, for example, at a base station receiver where users' codes and delays are known but channel parameters are not known. We believe that the present work is the first systematic investigation into the Taguchi experimental approaches for multiuser detection in DS-CDMA systems. Overall, the present Taguchi multiuser detectors will be shown to provide significant performance improvements over conventional DS-CDMA detectors and have performance close to that of MMSE detectors and decorrelating detectors [35] when they are applied at a base station.

This paper is organized as follows. In Section II, the DS-CDMA system model is presented. In Section III, the Taguchi method is described, and multiuser detection algorithms based on the Taguchi method are developed. In Section IV, simulation results are presented, which demonstrate the potential of the present multiuser detection scheme. Finally, several pertinent remarks are given in Section V to conclude this paper.

II. Received Signal Model and Notation

Consider a synchronous DS-CDMA system with K users, employing normalized spreading waveforms $s_1(t), \ldots, s_K(t)$, and transmitting sequences of binary phase-shift keying (BPSK) symbols through their respective multipath channels. The transmitted baseband signal during the ith symbol due to the kth user is given by

$$x_k(t) = A_k b_k(i) s_k(t - iT), \qquad k = 1, \ldots, K$$

where T is the symbol interval, A_k denotes the transmitted amplitude of the kth user and $b_k(i) = \pm 1$ is the ith transmitted symbol by the kth user and is assumed to be independent identically distributed (i.i.d.). It is assumed that $s_k(t)$ is supported only on the interval $[0, T]$. It is also assumed that user symbol sequences, $\{b_k(i),\ i = 1, 2, \ldots\}, k = 1, 2, \ldots, K$, from different users are independent. In DS-CDMA systems, the user spreading waveforms are of the form

$$s_k(t) = \sum_{j=0}^{N-1} c_k(j) \phi(t - jT_c), \qquad 0 \le t \le T$$

where N is the processing gain, ϕ is a normalized chip waveform of duration $T_c = T/N$, and $\mathbf{c}_k = [c_k(0), c_k(1), \ldots, c_k(N-1)]^{\mathrm{T}}$ is a signature chip sequence of $\pm 1/\sqrt{N}$ assigned to the kth user. Likewise, we shall assume i.i.d. random spreading sequences with

$$P\left[c_k(j) = \pm \frac{1}{\sqrt{N}}\right] = \frac{1}{2}.$$

Consider a slow fading channel with channel impulse response given by

$$h_k(t) = \sum_{l=1}^{L} g_{k,l} \delta(t - \tau_{kl})$$

where L is the number of paths in each user's channel, $g_{k,l}$ and τ_{kl} are, respectively, the real or complex gain and the delay of the lth path of the kth user's signal. It is assumed that τ_{kl} is an integer multiple of T_c [36] and is smaller than T [8], [17]. In this case, the received signal is given by

$$\begin{aligned} r(t) &= \sum_i \sum_{k=1}^{K} x_k(t) * h_k(t) + \nu(t) \\ &= \sum_i \sum_{k=1}^{K} \sum_{l=1}^{L} g_{k,l} x_k(t - \tau_{kl}) + \nu(t) \\ &= \sum_i \sum_{k=1}^{K} A_k b_k(i) \sum_{l=1}^{L} g_{k,l} \sum_{j=0}^{N-1} c_k(j) \\ &\quad \times \phi(t - jT_c - iT - \tau_{kl}) + \nu(t) \end{aligned}$$

where i is the index for information symbols and $\nu(t)$ is a white Gaussian noise with double-sided power spectral density of σ^2.

Note that, due to the channel dispersion, when each user transmits a sequence of information bits, there is intersymbol interference (ISI). For ISI suppression, we consider the use of "zero padding" as described in [2] and [32]. We assume that the path delays are given by $\tau_{kl} = (l-1)T_c$ for $l = 1, \ldots, L$ as in [8]. In the case when this assumption does not hold, i.e., when L is larger than the actual number of paths, we can consider to add some paths with zero gains so that the above assumption still holds. For example, if there are actually two paths in a system with $\tau_{k1} = 0$ and $\tau_{k2} = 3T_c$ and with $g_{k,1}$ and $g_{k,2}$ as the associated channel gains, we can add two paths with zero gains and with time delays $\tau'_{k2} = T_c$ and $\tau'_{k3} = 2T_c$, respectively. Thus, the number of paths becomes $L = 4$ and the path delays are 0, T_c, $2T_c$, and $3T_c$, respectively. The corresponding vector of channel gains is $[g'_{k,1}, g'_{k,2}, g'_{k,3}, g'_{k,4}]^{\mathrm{T}}$, where $g'_{k,1} = g_{k,1}$, $g'_{k,2} = g'_{k,3} = 0$ and $g'_{k,4} = g_{k,2}$. In this case, to eliminate the ISI, we need to use spreading code of length $P = N + L - 1$ to spread the user symbols. This redundancy is the key to avoiding ISI, as we will see below. The new spreading code for the kth user is given by

$$\mathbf{u}_k = [\, u_k(0), u_k(1), \ldots, u_k(P-1)\,]^{\mathrm{T}} \triangleq \mathbf{F_0}\mathbf{c}_k$$

where $\mathbf{F_0}$ is a $P \times N$ matrix (which will be determined later). In this case, the received signal is given by

$$y(t) = \sum_i \sum_{k=1}^{K} A_k b_k(i) \times \sum_{l=1}^{L} g_{k,l} \sum_{j=0}^{P-1} u_k(j)\phi(t - jT_c - iT - \tau_{kl}) + \nu(t).$$

At the receiver, we use chip-matched filtering followed by chip rate sampling to collect P measurements of $y(t)$ for the kth user during the ith symbol period and form a vector

$$\mathbf{y}_k = [\, y_k(0), y_k(1), \ldots, y_k(P-1)\,]^{\mathrm{T}}.$$

Channel dispersion gives rise to ISI between successive symbols and renders $\mathbf{y}_k$ dependent on both $b(i)$ and $b(i-1)$. In this case, $\mathbf{y}_k$ becomes

$$\mathbf{y}_k = \mathbf{H}_k^0 \mathbf{u}_k A_k b_k(i) + \mathbf{H}_k^1 \mathbf{u}_k A_k b_k(i-1) + \nu_k$$

where $\boldsymbol{\nu}_k$ is the noise vector corresponding to the kth user. The $P \times P$ matrices $\mathbf{H}_k^0$ and $\mathbf{H}_k^1$ are defined as follows:

$$\mathbf{H}_k^0 = \begin{pmatrix} g_{k,1} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ g_{k,2} & g_{k,1} & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ g_{k,L} & g_{k,L-1} & \cdots & \ddots & \cdots & \cdots & 0 \\ 0 & g_{k,L} & \cdots & \cdots & \ddots & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & g_{k,L} & \cdots & \cdots & g_{k,1} \end{pmatrix}$$

$$\mathbf{H}_k^1 = \begin{pmatrix} 0 & 0 & \cdots & g_{k,L} & \cdots & g_{k,3} & g_{k,2} \\ 0 & 0 & \cdots & 0 & \ddots & g_{k,4} & g_{k,3} \\ \vdots & \vdots & \ddots & \vdots & \cdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 & g_{k,L} \\ \vdots & \vdots & \cdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \end{pmatrix}.$$

The vector of equalized received data $\mathbf{r}_k$ is given by

$$\mathbf{r}_k = \mathbf{T}_0 \mathbf{y}_k$$

where $\mathbf{T}_0$ is an $N \times P$ matrix.

The ISI will be eliminated if we impose a structure on $\mathbf{F_0}$ and $\mathbf{T_0}$ so that

$$\mathbf{T}_0 \mathbf{H}_k^1 \mathbf{F}_0 = 0.$$

One choice for $\mathbf{F}_0$ and $\mathbf{T}_0$ is given by [2], [32]

$$\mathbf{F}_0 = \begin{bmatrix} \mathbf{I}_N \\ \mathbf{0}_{(L-1)\times N} \end{bmatrix} \text{and } \mathbf{T}_0 = [\, \mathbf{I}_N \quad \mathbf{0}_{N\times(L-1)}\,]$$

where $\mathbf{I}$ is an identity matrix and $\mathbf{0}$ is a matrix of zeros. The vector of equalized received data for the kth user can be written as

$$\begin{aligned} \mathbf{r}_k &= \mathbf{T}_0 \mathbf{y}_k \\ &= \mathbf{T}_0 \mathbf{H}_k^0 \mathbf{F}_0 \mathbf{c}_k A_k b_k(i) + \mathbf{T}_0 \mathbf{H}_k^1 \mathbf{F}_0 \mathbf{c}_k A_k b_k(i-1) + \mathbf{T}_0 \nu_k \\ &= \mathbf{T}_0 \mathbf{H}_k^0 \mathbf{F}_0 \mathbf{c}_k A_k b_k(i) + \mathbf{T}_0 \nu_k. \end{aligned}$$

We will rewrite $\mathbf{r}_k$ for each symbol of the kth user as

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{c}_k A_k b_k + \mathbf{T_0} \nu_k$$

where the $N \times N$ matrix

$$\mathbf{H}_k = \mathbf{T_0} \mathbf{H}_k^0 \mathbf{F_0} = \begin{pmatrix} g_{k,1} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ g_{k,2} & g_{k,1} & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ g_{k,L} & g_{k,L-1} & \cdots & \ddots & \cdots & \cdots & 0 \\ 0 & g_{k,L} & \cdots & \cdots & \ddots & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & g_{k,L} & \cdots & \cdots & g_{k,1} \end{pmatrix}.$$

We can also rewrite $\mathbf{r}_k$ as

$$\mathbf{r}_k = \mathbf{C}_k \mathbf{g}_k A_k b_k + \mathbf{T_0} \nu_k \quad (1)$$

where $\mathbf{g}_k = [g_{k,1}, \ldots, g_{k,L}]^{\mathrm{T}}$ is the channel gain vector for the kth user, $\mathbf{C}_k$ is the $N \times L$ spreading code matrix of the kth user defined by $\mathbf{C}_k \triangleq [\mathbf{c}_k^1 \;\vdots\; \mathbf{c}_k^2 \;\vdots\; \cdots \;\vdots\; \mathbf{c}_k^L]$, and $\mathbf{c}_k^l$, $k = 1, \ldots, K$, $l = 1, \ldots, L$, are acyclic shifted $\mathbf{c}_k$ with time shift indicated by τ_{kl} (i.e., τ_{kl}/T_c) [3], [17]. For example, if $\tau_{32} = 3T_c$, we have

$$\mathbf{c}_3^2 = [\,0,\ 0,\ 0,\ c_3(0),\ c_3(1), \ldots, c_3(N-4)\,]^{\mathrm{T}}.$$

Equation (1) holds since $\mathbf{H}_k\mathbf{c}_k = \mathbf{C}_k\mathbf{g}_k$ for all k. We can now determine the combined received signal within a symbol period T as [20]

$$\mathbf{r} = \mathbf{CGAb} + \mathbf{n} = \sum_{k=1}^{K} A_k b_k \sum_{l=1}^{L} g_{k,l} c_k^l + \mathbf{n} \tag{2}$$

where

$$\mathbf{C} \triangleq [\,\mathbf{C}_1, \ldots, \mathbf{C}_K\,] = \left[\,\mathbf{c}_1^1 \,\vdots\, \mathbf{c}_1^2 \,\vdots\, \cdots \,\vdots\, \mathbf{c}_1^L \,\vdots\, \cdots \,\vdots\, \mathbf{c}_K^1 \,\vdots\, \cdots \,\vdots\, \mathbf{c}_K^L\,\right]$$

is the $N \times KL$ data spreading code matrix. Matrix $\mathbf{G}$ in (2) is the block diagonal $KL \times K$ channel matrix defined by

$$\mathbf{G} = \mathrm{diag}\left(\begin{bmatrix} g_{1,1} \\ \vdots \\ g_{1,L} \end{bmatrix}, \ldots, \begin{bmatrix} g_{K,1} \\ \vdots \\ g_{K,L} \end{bmatrix}\right) = \begin{pmatrix} \begin{bmatrix} g_{1,1} \\ \vdots \\ g_{1,L} \end{bmatrix} & O & \cdots & O \\ O & \begin{bmatrix} g_{2,1} \\ \vdots \\ g_{2,L} \end{bmatrix} & \cdots & O \\ \vdots & \vdots & \ddots & \vdots \\ O & O & \cdots & \begin{bmatrix} g_{K,1} \\ \vdots \\ g_{K,L} \end{bmatrix} \end{pmatrix}$$

where each O is a matrix of zeros with appropriate dimension. Matrix $\mathbf{A}$ in (2) is a real $K \times K$ diagonal matrix of amplitudes of the transmitted signals, i.e., $\mathbf{A} = \mathrm{diag}[A_1, \ldots, A_K]$. We use $\mathbf{b} = [b_1, b_2, \ldots, b_K]^{\mathrm{T}}$ to denote the real K vector of input data symbols (typically BPSK) and $\mathbf{n}$ to denote the zero-mean Gaussian noise vector with i.i.d. components.

III. Taguchi Method for Multiuser Detection

A. Introduction to the Taguchi Method

For parameter optimization problems with a given computable objective function, the Taguchi method of experimental design [14] is a suitable method that can rapidly optimize the varying factors to get a desired outcome. Since the goal of this research is to maximize a likelihood function of optimal multiuser detector [30], it is appropriate to employ the Taguchi method. In the following, the Taguchi method will be described.

TABLE I
Initial Values of the Four Factors

Factor	Level 1	Level 2	Level 3
$h_{1,1}$	-0.3	0.1	0.5
$h_{1,2}$	-0.3	0.1	0.5
$h_{2,1}$	-0.3	0.1	0.5
$h_{2,2}$	-0.3	0.1	0.5

Suppose that an experimental outcome J is a cost function of several variables $h_{1,1}, h_{1,2}, \ldots, h_{m,n}$, whose values can be controlled. We write $J = J(h_{1,1}, h_{1,2}, \ldots, h_{m,n})$. The controlled variables $h_{k,l}$, $k = 1, \ldots, m$ and $l = 1, \ldots, n$, are called factors. The goal is to find the optimal values $\hat{h}_{k,l}$, $k = 1, \ldots, m$ and $l = 1, \ldots, n$, to minimize the cost function J. This can be done by varying the factors simultaneously in a disciplined manner and recording the corresponding values of J until we get the optimal $\hat{h}_{k,l}$, $k = 1, \ldots, m$ and $l = 1, \ldots, n$. The Taguchi method involves a disciplined method of varying two or more factors simultaneously.

In a full experimental design, all possible combinations of the values of factors must be tried. In a fractional design, such as the Taguchi method, a subset of the possible value combinations is used. To reduce the time consumed in conducting experiments while taking advantage of the performance of full factorial method, the Taguchi method based on orthogonal arrays was introduced. It is a method of setting up experiments that only requires a fraction of the full factorial combinations. The experiment combinations are chosen to provide sufficient information to determine the effects of each factor.

We illustrate next an example of design involving four factors. The four factors are denoted by $h_{1,1}$, $h_{1,2}$, $h_{2,1}$, and $h_{2,2}$, and Table I gives the three initial values (which are called levels) for each of the four factors. Generally speaking, these initial values are selected randomly in an ascending order, i.e., level $1 <$ level $2 <$ level 3.

We will use the orthogonal array shown in Table II for the purpose of demonstration. In the present example, each factor has three different levels, and they are denoted by $h_{k,l}^{(1)} = -0.3$, $h_{k,l}^{(2)} = 0.1$, and $h_{k,l}^{(3)} = 0.5$ for k, $l = 1, 2$. If we use the full factorial method to discover the optimal combination of these factors, we need to conduct $3^4 = 81$ tests, whereas the orthogonal array $L_9(3^4)$ in Table II allows us to set up experiments with only nine tests. The orthogonal array in Table II is in the form of $L_M(q^m)$, where q is the number of levels each factor has, m is the maximum number of factors the table can handle, and M is the total number of tests required using this table. In general, M is much smaller than the value of q^m, which is the total number of combinations for m factors with each having q levels (choices). A cycle in the present Taguchi method is defined as a complete set of tests according to the orthogonal array, consisting of a total of M tests. In an orthogonal array (e.g., Table II), the numbers under each factor in a test indicate the level of the factor to be used in the test. For example, in test number 4, we would use: $h_{1,1}^{(2)}$ (level 2 of $h_{1,1}$); $h_{1,2}^{(1)}$ (level 1 of $h_{1,2}$); $h_{2,1}^{(2)}$ (level 2 of $h_{2,1}$); and $h_{2,2}^{(3)}$ (level 3 of $h_{2,2}$). Orthogonal arrays are readily composed and are available from

TABLE II
ORTHOGONAL ARRAY $L_9(3^4)$

Test Number	$h_{1,1}$	$h_{1,2}$	$h_{2,1}$	$h_{2,2}$	Cost
1	1	1	1	1	J_1
2	1	2	2	2	J_2
3	1	3	3	3	J_3
4	2	1	2	3	J_4
5	2	2	3	1	J_5
6	2	3	1	2	J_6
7	3	1	3	2	J_7
8	3	2	1	3	J_8
9	3	3	2	1	J_9
Contributions of level 1	$V_{1,1}^{(1)}$	$V_{1,2}^{(1)}$	$V_{2,1}^{(1)}$	$V_{2,2}^{(1)}$	
Contributions of level 2	$V_{1,1}^{(2)}$	$V_{1,2}^{(2)}$	$V_{2,1}^{(2)}$	$V_{2,2}^{(2)}$	
Contributions of level 3	$V_{1,1}^{(3)}$	$V_{1,2}^{(3)}$	$V_{2,1}^{(3)}$	$V_{2,2}^{(3)}$	

many texts (e.g., [4], [7]). The way that they are constructed [7] is to have each level of every factor appear the same number of times in every column of the array (e.g., three times in Table II), and each combination of factors between any two columns, i.e., each (i, j), $i, j = 1, 2, 3$, appears the same number of times [e.g., each pair (i, j) between every two columns in Table II appear one time].

Using the orthogonal array $L_9(3^4)$, each cycle consists of nine individual tests. After each cycle of tests, a minimum cost can be found. While this cost may not be the optimal cost, more cycles are needed until the minimum cost of each cycle converges.

In the present example with four factors, after each cycle of tests, we perform an analysis to determine the trend of the cost function for each factor. The values of the cost function from the nine tests are calculated and denoted by J_i, $i = 1, 2, \ldots, 9$. For each of the four factors, we calculate the total contribution of each level to the cost function $V_{k,l}^{(j)}$ as the sum of the cost values corresponding to the tests involving that particular level. For example, after nine tests are completed, for factor $h_{2,1}$, we calculate

$$\begin{aligned} V_{2,1}^{(1)} &= J_1 + J_6 + J_8 \\ V_{2,1}^{(2)} &= J_2 + J_4 + J_9 \\ V_{2,1}^{(3)} &= J_3 + J_5 + J_7 \end{aligned} \tag{3}$$

where $V_{k,l}^{(j)}$ indicates the total contribution of the jth level of the factor $h_{k,l}$ to the cost function. $V_{2,1}^{(1)}$ is the summation of J_1, J_6, and J_8 since the three tests involving the first level of $h_{2,1}$ are test numbers 1, 6, and 8. We will then have three total contributions that correspond to the three levels for each factor calculated according to Table II. These three total contributions can be plotted versus the three levels for each factor to determine the trend of the cost function as shown in Fig. 1. In this figure, the numbers "1," "2," and "3" along the horizontal axis represent the three levels of each factor, and $V^{(1)}$, $V^{(2)}$, and $V^{(3)}$ are the total contributions of each level of a factor.

From these figures for the trend of the cost function, we know whether we need to increase or decrease the value for each factor. If the trend of the cost function is as shown in Fig. 1(a) or (b), this means that the value of this factor should be increased in order to further reduce the value of the cost function. In this case, we can choose a step size (e.g., 0.01) and increase all three levels of the factor by the chosen step size. Alternatively, we can also use the estimated gradient (see Appendix) information to determine the direction for each factor to move in and the amount to adjust. Likewise, for trend as shown in Fig. 1(d) or (e), the parameter values should be decreased. In the Appendix, we will show that the gradient of the cost function J with respect to the factor $h_{k,l}$ can be estimated from the experiments using

$$\nabla_{k,l} J = \frac{3}{2M} \left(V_{k,l}^{(3)} - V_{k,l}^{(1)} \right) \frac{1}{\delta}$$

where $M = 9$ and $\delta = 0.4$ in the present example. This indicates that if the trends of cost function from the experiments are given as in Fig. 1(a), (b), (d), or (e), we have a very good estimate of the gradient of the cost function. Note that to minimize a cost function, the key is to determine its gradient with respect to varying parameters. If the trend is as shown in Fig. 1(c), this means that the parameter value should be set closer to the middle level or the center of the parabolic curve. In case of Fig. 1(f), we can randomly select a direction, which means either increase or decrease the parameter value. The analysis based on Fig. 1(c) and (f) implies that using three levels for each parameter in the experiments will provide better results than using two levels as in [31]. If two levels for each parameter are used in the experiments, for cases as shown in Fig. 1(c) and (f), the decisions about the next move in the parameter space will often be incorrect. This is especially true for the case as shown in Fig. 1(c). The analysis in Fig. 1(c) of the experimental results indicates that we should stay in the neighborhood of the current value and shrink the interval of search toward the center of the parabolic curve. From the three points shown in Fig. 1(c), we can fit a parabolic function between the cost values and the factor values using

$$V = \alpha x^2 + \beta x + \gamma.$$

The coefficients α, β, and γ can easily be determined by plugging the values of $(x^{(1)}, V^{(1)})$, $(x^{(2)}, V^{(2)})$, and $(x^{(3)}, V^{(3)})$ into the above expression, where $x^{(i)}$ is the value of level i in Fig. 1(c). The center of the fitted parabolic curve will be chosen as the new $x^{(2)}$, i.e., $x_{\text{new}}^{(2)} = -\beta/(2\alpha)$. The new values of level 1 and level 3 are chosen as $x_{\text{new}}^{(1)} = x_{\text{new}}^{(2)} - 0.5\eta(x^{(3)} - x^{(1)})$ and $x_{\text{new}}^{(3)} = x_{\text{new}}^{(2)} + 0.5\eta(x^{(3)} - x^{(1)})$, where η is a shrinking coefficient (e.g., $\eta = 0.9$).

According to the trend determined for each factor, we choose a new set of three initial levels for each factor, and we start a new cycle and perform again nine tests according to Table II. The procedure is repeated until the value of the cost function is converged. To determine the convergence of the present iterative algorithm, we compute the minimum cost in each cycle. Define

$$J_{\min}(p) = \min_{i=1,\ldots,M} \{J_i(p)\}$$

where M is the number of tests in each cycle according to the orthogonal array, $J_i(p)$ is the cost for the ith test in the

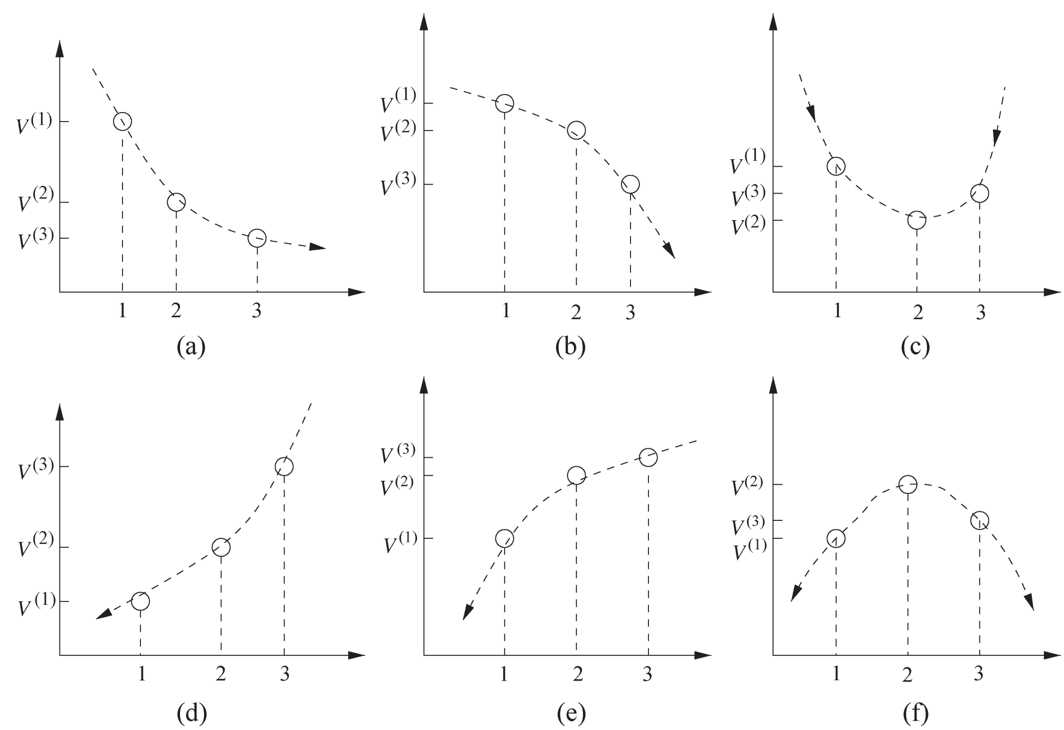

Fig. 1. Six different trends of the cost function.

pth cycle. The difference between the $J_{\min}$ of two consecutive cycles of the Taguchi method will be used to decide whether to stop the algorithm. In particular, the convergence criterion in the present case is $J_{\min}(p-1) - J_{\min}(p) \leq 10^{-2}$, i.e., when the improvement of the cost function from one cycle to the next is less than 10^{-2}, we stop the algorithm. Clearly, such a procedure will achieve the minimization of the cost function $J(h_{1,1}, h_{1,2}, h_{2,1}, h_{2,2})$ through repeated cycles of tests and analyses according to Table II and Fig. 1. Such a method based on orthogonal arrays is usually referred to as the Taguchi method [4], [14], [25], which we will use for multiuser detection in DS-CDMA wireless systems.

B. Taguchi Multiuser Detection in Complex Channels

In this section, the problem of multiuser detection for DS-CDMA systems is solved using the Taguchi method. We consider the case of complex channels where parameters do not change in a period of information frame.

The maximum likelihood multiuser detector for single path channels was derived in [18] and [29]. The extension to multipath channels was presented in [35] to obtain optimal multiuser detector as follows: From (2), the jointly optimum detector finds the symbol vector estimate $\mathbf{b} = [b_1, \ldots, b_K]^{\mathrm{T}}$, which maximizes the likelihood function given by [30]

$$\exp\left[-\left(\frac{1}{2\sigma^2}\right)\|\mathbf{r} - \mathbf{CGAb}\|^2\right].$$

This is equivalent to minimizing the negative of the exponential function's argument, which in turn is equivalent to minimizing

$$J = -2\mathrm{Re}(\mathbf{b}^{\mathrm{T}}\mathbf{A}\mathbf{G}^{\mathrm{H}}\mathbf{C}^{\mathrm{T}}\mathbf{r}) + \mathbf{b}^{\mathrm{T}}\mathbf{A}\mathbf{G}^{\mathrm{H}}\mathbf{C}^{\mathrm{T}}\mathbf{CGAb}. \quad (4)$$

Since the second term does not depend on the received signal, if the channel is not known, a vector of sufficient statistics [20] can be formed at the output of the code matched filters as

$$\mathbf{y} \triangleq \mathbf{C}^{\mathrm{T}}\mathbf{r} = \mathbf{C}^{\mathrm{T}}\mathbf{CGAb} + \mathbf{C}^{\mathrm{T}}\mathbf{n} = \mathbf{Rh} + \lambda \quad (5)$$

where $\mathbf{R} \triangleq \mathbf{C}^{\mathrm{T}}\mathbf{C}$ is the spreading code correlation matrix, $\lambda \triangleq \mathbf{C}^{\mathrm{T}}\mathbf{n}$ is the resulting noise vector, and $\mathbf{h} \triangleq \mathbf{GAb}$. Getting the optimal $\mathbf{b}$ that minimizes (4) is equivalent to obtaining an $\mathbf{h}$ given by

$$\begin{aligned}\mathbf{h} &= [h_{1,1}, \ldots, h_{1,L}, h_{2,1}, \ldots, h_{K,1}, \ldots, h_{K,L}]^{\mathrm{T}} \\ &= [g_{1,1}A_1b_1, \ldots, g_{1,L}A_1b_1, g_{2,1}A_2b_2, \ldots, \\ &\qquad g_{K,1}A_Kb_K, \ldots, g_{K,L}A_Kb_K]^{\mathrm{T}}\end{aligned}$$

which minimizes the following cost function

$$-2\mathrm{Re}(\mathbf{h}^{\mathrm{H}}\mathbf{y}) + \mathbf{h}^{\mathrm{H}}\mathbf{Rh}. \quad (6)$$

Note that (6) is equivalent to (4). Since $h_{k,l} \triangleq g_{k,l}A_kb_k$, without loss of generality, we assume that all users have equal unit transmitted power, i.e., $A_1 = \cdots = A_K = 1$. With this assumption, we have $h_{k,l} = \pm g_{k,l}$ since $b_k = \pm 1$. The received signal powers will be unequal due to the unequal strength of the multipath gain $g_{k,l}$ for each user. The optimal multiuser detection algorithm calculates the estimated $\hat{\mathbf{h}}$ by minimizing (6)

$$\hat{\mathbf{h}} = \arg\min_{\mathbf{h}} \left\{-2\mathrm{Re}(\mathbf{h}^{\mathrm{H}}\mathbf{y}) + \mathbf{h}^{\mathrm{H}}\mathbf{Rh}\right\}. \quad (7)$$

In (7), $\mathbf{h} = [h_{1,1}, \ldots, h_{1,L}, h_{2,1}, \ldots, h_{K,1}, \ldots, h_{K,L}]^{\mathrm{T}}$ and $h_{k,l}$ stands for the candidates of the estimated $\hat{\mathbf{h}}$ and $\mathbf{y} = [y_{1,1}, \ldots, y_{1,L}, \ldots, y_{K,1}, \ldots, y_{K,L}]^{\mathrm{T}}$ stands for the multiuser spreading code matched filter output given by (5).

We can see that the minimization of (6) can be achieved by using

$$\hat{\mathbf{h}} = \mathbf{R}^{-1}\mathbf{y} \tag{8}$$

because

$$-2\mathrm{Re}(\mathbf{h}^{\mathrm{H}}\mathbf{y}) + \mathbf{h}^{\mathrm{H}}\mathbf{R}\mathbf{h} = -\mathbf{y}^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{y} + (\mathbf{h} - \hat{\mathbf{h}})^{\mathrm{H}}\mathbf{R}(\mathbf{h} - \hat{\mathbf{h}}).$$

The detector in (8) is known as the decorrelating detector. In this case, we have to assume that the spreading code correlation matrix is nonsingular. On the other hand, the inversion of the matrix $\mathbf{R}$ may be computationally prohibitive in systems with large number of users. Since $\mathbf{R} = \mathbf{C}^{\mathrm{T}}\mathbf{C}$ and $\mathbf{C}$ is an $N \times KL$ matrix, the matrix $\mathbf{R}$ is not invertible when $KL > N$. To avoid calculating the inverse of the correlation matrix $\mathbf{R}$, other approaches can be considered. After $\mathbf{h}$ is determined as

$$\hat{\mathbf{h}} = [\,\hat{h}_{1,1}, \ldots, \hat{h}_{1,L}, \ldots, \hat{h}_{K,1}, \ldots, \hat{h}_{K,L}\,]^{\mathrm{T}}$$

assuming that the transmitted bit sequence is differentially encoded, we follow the procedure in [8] and [21] to get the suboptimal estimate $\hat{\mathbf{b}} = [\hat{b}_1, \ldots, \hat{b}_k]$ for the ith transmitted bit as

$$\hat{b}_k(i) = \mathrm{sgn}\left\{\sum_{l=1}^{L} \mathrm{Re}\left[\hat{h}_{k,l}(i)\hat{h}^{*}_{k,l}(i-1)\right]\right\}, \qquad k = 1, \ldots, K$$

which in fact is a noncoherent equal gain RAKE combiner [8], [22].

Since $\mathbf{y}$ and $\mathbf{R}$ are known in (6), we can use the Taguchi method to obtain a solution that minimizes the cost function in (6). The function in (6) is therefore chosen as our cost function in the Taguchi method described earlier. The number of factors to be determined depends on the product of the number of users K and the number of multipath components L. Note that, compared to real channels, the number of factors will be doubled in complex channels, because for each parameter $h_{k,l}$, both real and imaginary parts, will be determined. The real part and the imaginary part of $h_{k,l}$ will be treated as two independent factors using the Taguchi method, i.e., each complex parameter $h_{k,l}$ will occupy two columns in the orthogonal array. Each factor is with three levels, and thus three initial values have to be set for each factor. Without loss of generality, we set the same initial levels to all factors. Then we repeat the experiments after adjusting the three levels according to the trend of the cost function, until a satisfactory solution is obtained, which corresponds to the case where one of the three levels is close to or equal to the optimal solution.

As with most experimental methods for optimization, there is a concern that the present Taguchi method may also get stuck in local minima in some cases. To avoid local minimum solutions, we rotate the orthogonal array so that different variations of the array are used in different cycles. In the first cycle, we use the array as it is, e.g., as in Table II. In the next cycle, we move the second column of the orthogonal array to the first, the third column to the second,. . ., and the first column to the last, so that different level combinations are tested for each factor in different cycles. In this way, we can reduce the possibility of getting stuck in local minima. When the orthogonal array is rotated from one cycle to the next, different factors will be used in different dimensions when tests are designed according to the array. We will use an example to show the effect of such rotations in Section IV (cf. Example 2).

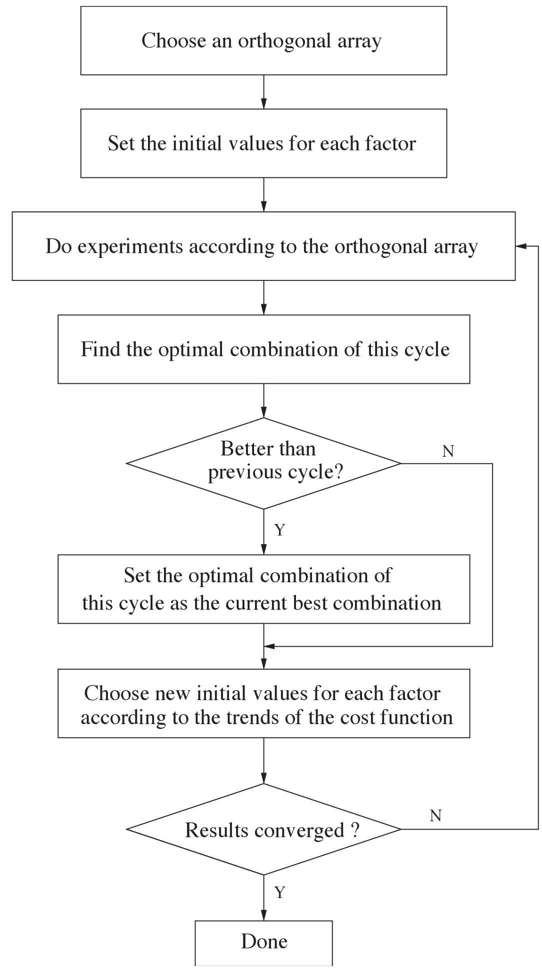

Fig. 2. Flow diagram of the Taguchi multiuser detection algorithm.

From the discussion of the Taguchi method above, a flow diagram of the Taguchi multiuser estimator is shown in Fig. 2. When $h_{k,l}$ is complex, such procedure will be applied to both the real part and the imaginary part of $h_{k,l}$.

C. Taguchi Multiuser Detection in Real Channels

In this subsection, we consider applying the above procedure to real channels. In particular, we consider the case where the channel gains are real and positive [6], [30]. Taking into account that $\mathbf{r}$ and $\mathbf{G}$ are real variables, we have the following function to minimize:

$$J = -2(\mathbf{b}^{\mathrm{T}}\mathbf{A}\mathbf{G}^{\mathrm{T}}\mathbf{C}^{\mathrm{T}}\mathbf{r}) + \mathbf{b}^{\mathrm{T}}\mathbf{A}\mathbf{G}^{\mathrm{T}}\mathbf{C}^{\mathrm{T}}\mathbf{C}\mathbf{G}\mathbf{A}\mathbf{b}.$$

Similarly, a vector of sufficient statistics can be formed at the output of the code matched filters as

$$\mathbf{y} \triangleq \mathbf{C}^{\mathrm{T}}\mathbf{r} = \mathbf{C}^{\mathrm{T}}\mathbf{CGAb} + \mathbf{C}^{\mathrm{T}}\mathbf{n} = \mathbf{Rh} + \lambda.$$

Thus, we have the following cost function [cf. (6)] to minimize:

$$-2\mathbf{h}^{\mathrm{T}}\mathbf{y} + \mathbf{h}^{\mathrm{T}}\mathbf{Rh}.$$

We still assume that all users have equal unit transmitted power. Under this assumption, we have $h_{k,l} = \pm g_{k,l}$, since $b_k = \pm 1$. Similarly, the suboptimal estimate $\hat{\mathbf{b}} = [\hat{b}_1, \ldots, \hat{b}_k]$ for the ith transmitted bit can be determined by

$$\hat{b}_k(i) = \mathrm{sgn}\left\{\sum_{l=1}^{L}\left[\hat{h}_{k,l}(i)\hat{h}_{k,l}(i-1)\right]\right\}, \qquad k = 1, \ldots, K$$

assuming that the transmitted bit sequence is differentially encoded. We follow the same procedure as described in the previous subsection to apply the Taguchi method to multiuser detection.

D. Advantages of the Taguchi Multiuser Detection Method

The Taguchi method provides us with a systematic and efficient method for conducting experimentations to determine near optimum values of the controllable factors $h_{k,l}$. By using orthogonal arrays, the Taguchi method searches in the parameter space with a small number of experiments. The savings will be greater when the number of factors in the problem is larger. For example, in our simulations, by using an array $L_{27}(3^{22})$, 20 factors each with three levels are optimized by running only 27 tests as opposed to a total of $3^{20} \approx 3.4868 \times 10^9$ required by a full experimental design in each cycle. It will be shown in Section IV that the multiuser detector developed in this paper reaches convergence within 12 cycles, which implies that only a total of 324 $(= 27 \times 12)$ iterations are needed to obtain the final satisfactory solution.

There are several important features of the Taguchi method developed in this paper. They are enumerated below.

1) *No channel estimation required*. The present Taguchi method is blind since no channel estimation is required to detect the transmitted data, which is not the case in many existing methods.
2) *Insensitivity to the choice of initial values of parameters* $h_{k,l}$. In most algorithms, if the initial values are far away from the optimal value, their complexities increase greatly. Thus, how to choose initial values that are close to the optimal values is a big concern to many existing algorithms. The Taguchi method is insensitive to the choice of initial values, which eases this concern. In Section IV, we will show using an example that the present multiuser detector is insensitive to the choice of initial values of $h_{k,l}$. Using the present algorithm, the randomly chosen initial levels $(h_{k,l}^{(1)}, h_{k,l}^{(2)}, h_{k,l}^{(3)})$ may or may not cover the actual value of $h_{k,l}$ in its range, i.e., it may happen that $h_{k,l}^{(j)} > h_{k,l}$ for all j or $h_{k,l}^{(j)} < h_{k,l}$ for all j. Simulation results reveal that the performance of the present algorithm is insensitive to the choice of initial levels used in the experiments.
3) *Easy implementation and fast convergence*. The calculations of the values of the cost function J in (4) [or in (6)] are straightforward given all the required information. In Section IV, we will show that the present algorithm for multiuser detection has a very fast convergence speed. Furthermore, the more complicated the cost function, the more obvious this advantage is.
4) *Suitability for parallel implementation*. In every cycle of tests in the present algorithm, the computations required for all the tests can be done in parallel on different processors. For example, in the case of using the orthogonal array $L_9(3^4)$, the nine tests in every cycle compute the values of the cost function J. In these computations, the structure of the detector remains the same, and we only choose to test different value combinations of parameters. Therefore, these computations can be done in parallel to achieve even faster convergence.
5) *No constraints on parameters (factors)*. The present algorithm does not rely on any imposed constraints, as opposed to, e.g., techniques described in [27] and [36], whose performance may be sensitive to the satisfaction of some constraints on parameters.

IV. Simulation Results

In this section, we assess the performance of the Taguchi multiuser detector developed in the previous section using computer simulations. As opposed to the solutions considered in [6], the present technique does not require the knowledge of channel parameters, including the fading coefficients. However, we still assume the knowledge of users' codes and delays since we assume that we use the present Taguchi multiuser detectors at a base station. Throughout this section, TMUD shall refer to the present Taguchi multiuser detector. Synchronous DS-CDMA transmission in multipath environments will be considered in our simulation studies. The simulated systems used in Examples 1 and 2 consist of five users $(K = 5)$ with a spreading gain 31 $(N = 31)$. The systems in Examples 3 and 4 consist of ten users $(K = 10)$. Each user's propagation channel consists of two paths. Our simulation results are obtained by averaging over 500 independent runs in each example. In each run, channel gains are generated randomly using zero-mean Gaussian distribution with variance given by $1/L$ (L is the number of paths) and multipath delays are generated using uniform distribution in $[1, 6T_c)$ [17]. A total of 1000 symbols are randomly generated and transmitted in each run. Furthermore, the additive noise is generated using zero-mean Gaussian distribution with appropriate variance depending on the required SNR in each run of our simulation. The three initial levels of each factor $h_{k,l}$ (real part and imaginary part in Examples 1 and 2) are chosen as −0.5, 0.1, and 0.5, unless indicated otherwise. Finally, the long code (random code) sequence [6] is used in our examples. In all examples, we detect all the users in the system unless indicated otherwise. We use a

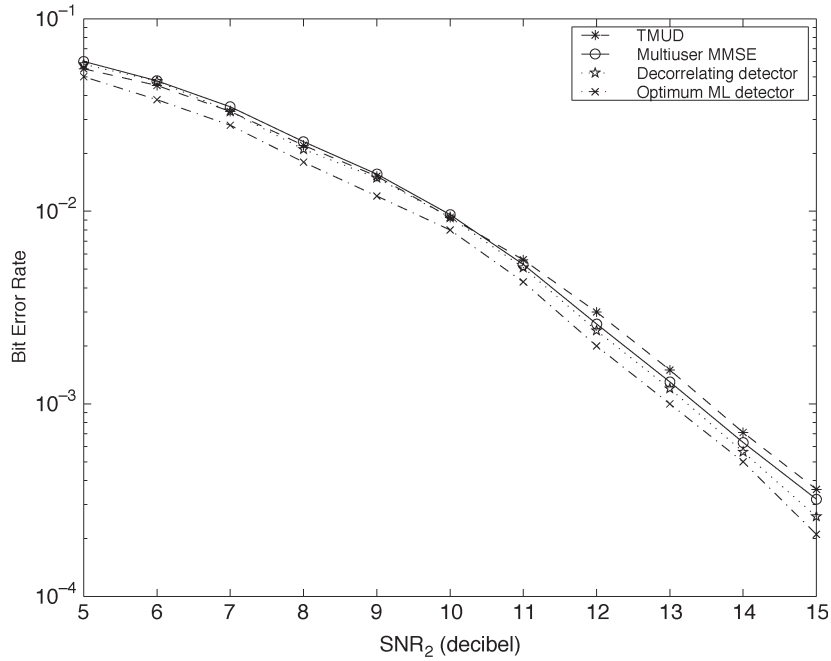

Fig. 3. Comparison of bit error rate versus SNR for four detectors over a complex frequency-selective Rayleigh slow-fading channel: TMUD, multiuser MMSE, decorrelating detector, and optimum ML detector. $\mathrm{SNR}_i/\mathrm{SNR}_2$ is fixed at 6 dB $(i = 1, 3, 4, 5)$.

fixed step size of 0.01 and $\eta = 0.9$ for parameter updates in our examples.

Example 1: The system used in the present example consists of five users $(K = 5)$. Each user's propagation channel is a complex frequency-selective Rayleigh slow-fading channel, and each consists of two paths. The channel parameters used in this example are generated using the approach in [36]. The channel parameters for user number 2 are obtained by sampling the following function generated from a two-ray Rayleigh fading model [23], [36]

$$h(t, \tau) = A\alpha_1(t)\delta(\tau) + B\alpha_2(t)\delta(\tau - \tau_1)$$

where $\alpha_1(t)$ and $\alpha_2(t)$ are two independent complex Gaussian processes with unit power, and τ_1 is the delay between the two paths. For user number 2, we choose $A = 0.4$ and $B = 0.7$. Other user parameters are obtained with different values of A and B. Because in a complex case, unknown factors are doubled, the same orthogonal array used in the complex case can handle twice as many users as in the real case. The present simulation results are shown in Fig. 3. In this example, we compare the present TMUD with the well-known multiuser detection techniques, i.e., the linear multiuser MMSE detector [35], the linear decorrelating detector [35], and the optimum maximum likelihood detector using the Viterbi algorithm. There are five users in the system. User number 2 is the weakest. The other four users have equal power, and $\mathrm{SNR}_i/\mathrm{SNR}_2 = 6$ dB, $i = 1, 3, 4, 5$, is fixed in the simulation. In Fig. 3, we can see that the performance of TMUD, MMSE, and decorrelating detectors are close to each other. We also noticed that the computational complexity of the Taguchi method is comparable to that of MMSE and decorrelating detectors. In this case, the multiuser MMSE detector [35], the linear decorrelating detector [35], and the optimum maximum likelihood detector assume that at the base station, it is possible to know the spreading sequences and delays as well as channel gains of all users, while the present TMUD assumes no knowledge about users' channel parameters. We record the minimum value of the cost function in each cycle. If the minimum cost value from one cycle to the next does not reduce by more than 0.01, we stop the algorithm and we consider that the algorithm has converged. In this example and in all examples to follow, our algorithm converged within 12 cycles of tests.

Example 2: In this example, we will show the effects of rotating the orthogonal array on the performance of TMUD. The system is the same as the one used in Example 1. The orthogonal array will be rotated from one cycle of tests to the next. In the first cycle, we use the orthogonal array as it is. In the next cycle, we move the second column of the orthogonal array to the first, the third column to the second,. . ., and the first column to the last. After each cycle of tests, we perform the rotation again in the same way, so that different level combinations are tested for each factor in different cycles. In Fig. 4, we can see that with the help of rotating the orthogonal array, the Taguchi method can avoid local minima and results in much better performance.

Example 3: The system used in the present example consists of 10 users $(K = 10)$. Each user's propagation channel is real and consists of two paths. Fig. 5 compares the performance of our algorithm and the optimum maximum likelihood detector, the decorrelating detector, and the conventional detector (i.e., matched filter detector) over a real frequency-selective Rayleigh slow-fading channel. We can see that the present TMUD significantly outperforms the conventional receiver, and the performance of TMUD is very close to that of the decorrelating detector. We note that in the results shown in Fig. 5, both the optimum maximum likelihood detector and the

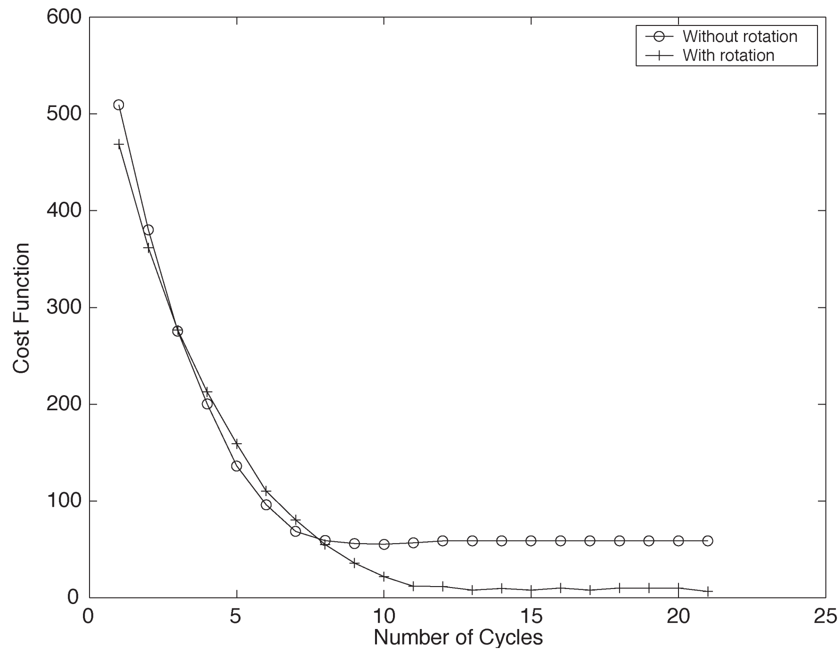

Fig. 4. Effect of rotating the orthogonal array on the performance of the Taguchi multiuser detector.

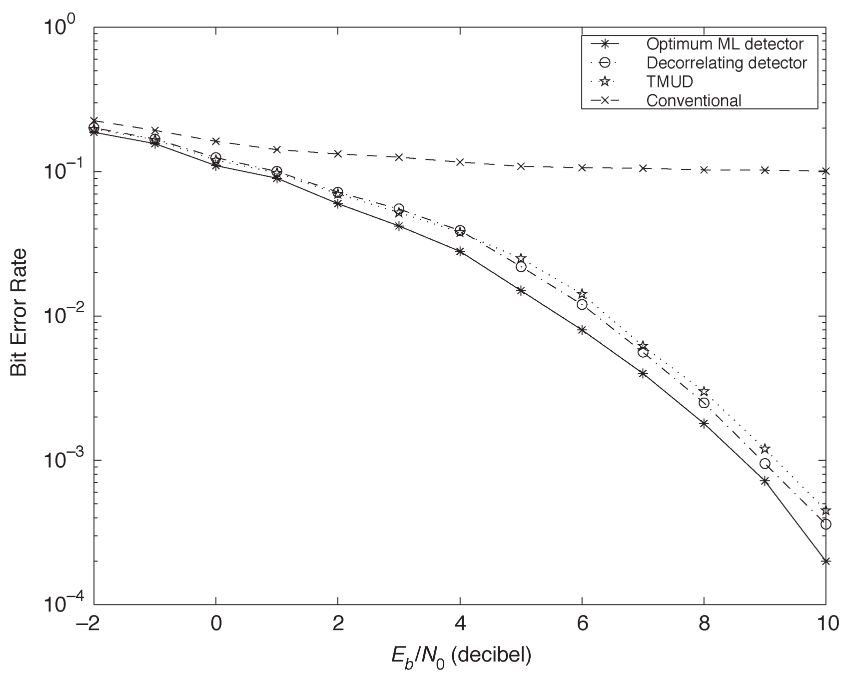

Fig. 5. Bit error rate versus E_b/N_0 with 10 users for a multipath synchronous DS-CDMA system over a real frequency-selective Rayleigh slow-fading channel.

decorrelating detector require the knowledge of channel gains, while our TMUD assumes no knowledge about channel gains.

Example 4: Recall that in Section III, we mentioned that the Taguchi method is insensitive to the choice of initial values. We next consider the system performance with different choices of initial values. Two extreme cases are compared to the case that we used in Example 3: Case 1: the initial three levels are all negative; and Case 2: the initial three levels are all positive. Some of the choices for initial levels in the two cases could be totally wrong, which may be considered as due to an incorrect guess of the range of the actual $h_{k,l}$. In Fig. 6, we can see that the system performance is very close to each other no matter what initial levels we choose. Fig. 7 displays the number of cycles needed for convergence for different choice of initial values, we can see that no matter what initial values are set, the TMUD converges after approximately 12 cycles of tests.

Example 5: In this example, we illustrate how performance varies with the number of users for both the multiuser MMSE detector proposed in [35] and the present TMUD. The simulated system has real channel parameters in this case and has a processing gain of $N = 15$. The number of paths for each user is two. The performance of the proposed MMSE

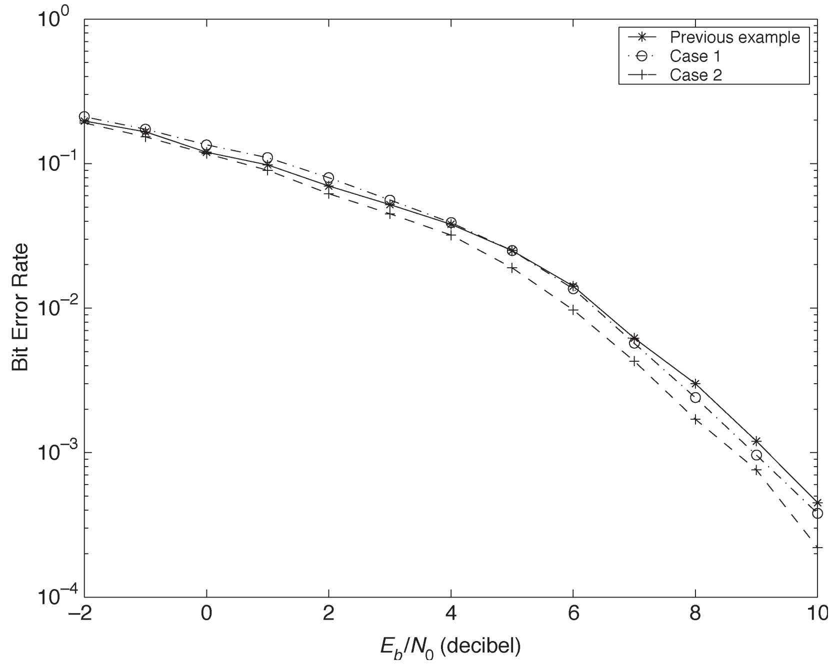

Fig. 6. Bit error rate versus E_b/N_0 with 10 users for a multipath synchronous DS-CDMA system over a real frequency-selective Rayleigh slow-fading channel. Case 1: the initial three levels of parameters are all negative. Case 2: the initial three levels are all positive.

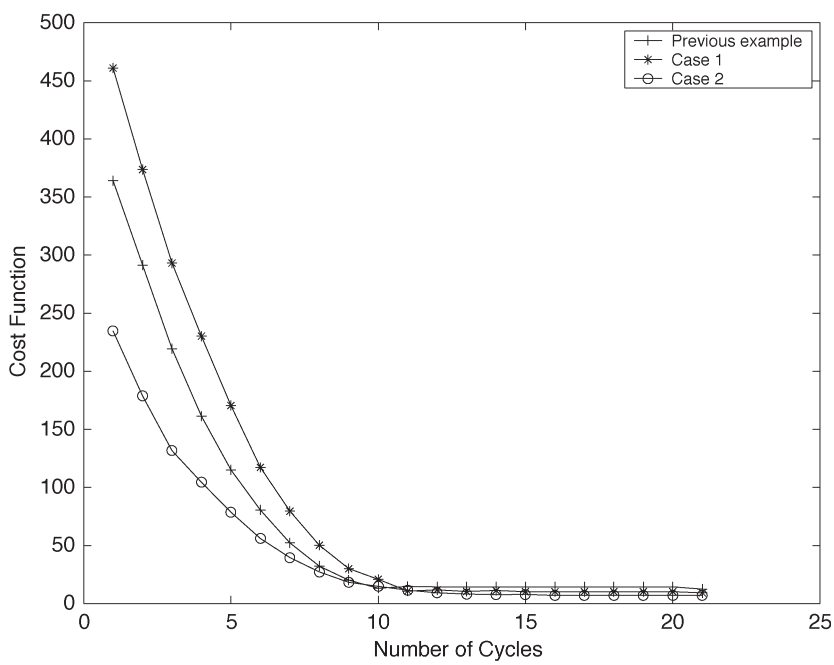

Fig. 7. Convergence behavior under different choices of initial levels. Case 1: the initial three levels are all negative. Case 2: the initial three levels are all positive.

in [35] and the present TMUD for user number 2 is plotted, respectively, in Fig. 8, with the number of users $K = 10$ and 20, respectively. In the figure, we can see that in both cases, the performance of TMUD is close to that of the MMSE detector.

V. Conclusion

In this paper, we considered the problem of multiuser detection in multipath DS-CDMA channels. The Taguchi multiuser detectors have been studied in synchronous multipath channels without the knowledge of channel information. Our analysis and numerical simulation results show that the present Taguchi multiuser detectors are insensitive to channel noise and insensitive to the choice of initial values for parameters. Another advantage of the present Taguchi method is that it is blind since no channel estimation is required to detect the transmitted data, which is not the case in many existing methods. Combined with their relatively low computational requirements as well as their suitability for parallel implementation, we believe that

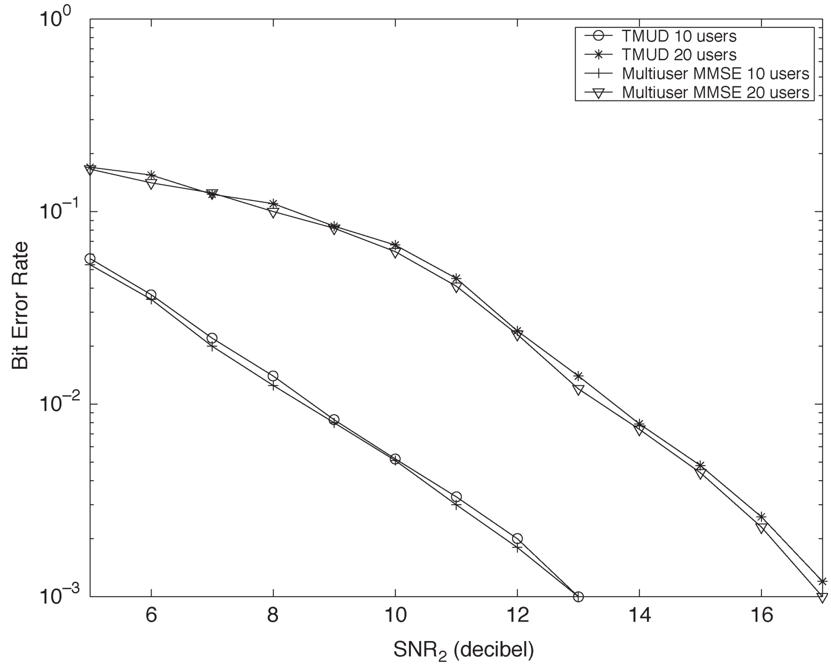

Fig. 8. Multiuser detector performance for user number 2 under different number of users ($N = 15$, $L = 2$).

these features make the present Taguchi multiuser detectors a viable option for improving the capacity of wireless DS-CDMA systems.

Appendix

In this Appendix, we will show that the gradient of a cost function can easily be estimated from experiments designed according to orthogonal arrays. In particular, we confirm the cases of trends considered in Fig. 1 used in our algorithm for maximizing the likelihood function in multiuser detection. We note that a previous proof has been provided in [31] for the Taguchi designs using two levels, and we provide our proof for three levels for completeness of our results. We also point out that the use of three levels in the Taguchi method is more efficient than the use of two levels.

Assume that a cost function J is a complicated nonlinear function of m factors, $J = J(\eta_1, \eta_2, \ldots, \eta_m)$. Here, η_j, $j = 1, \ldots, m$ are equivalent to the factors $h_{k,l}$, $k = 1, \ldots, K$ and $l = 1, \ldots, L$, described in Section III (i.e., we have $m = KL$). Choose the initial point as $\eta^0 = (\eta_1^0, \eta_2^0, \ldots, \eta_m^0)$ and small interval as $\Delta = (\delta_1, \delta_2, \ldots, \delta_m)$. Define

$$\eta_j^{(1)} = \eta_j^0 - \delta_j \quad \eta_j^{(2)} = \eta_j^0 \quad \eta_j^{(3)} = \eta_j^0 + \delta_j$$

where $\eta_j^{(1)}$ is the lower level (level 1) of η_j, $\eta_j^{(2)}$ is the middle level (level 2) of η_j, and $\eta_j^{(3)}$ is the upper level (level 3) of η_j.

To use the Taguchi method, all tests are designed according to the orthogonal array, for example, as in Table II. We assume the use of orthogonal array $L_M(3^m)$, where M is the required number of tests in the orthogonal array for each cycle of tests and m is the total number of columns of the orthogonal array. Define a matrix $X = [x_{ij}] \in R^{M \times m}$, where $x_{ij} = -1$ corresponds to level 1 in the orthogonal array, $x_{ij} = 0$ corresponds to level 2, and $x_{ij} = 1$ corresponds to level 3. An orthogonal array has the following properties [7]: 1) For every factor, each level appears the same total number of times in each cycle of tests. For example, we can see in Table II that each level appears three times in each cycle of tests for every factor. 2) All factors' different combinations of levels between any two columns appear the same number of times in the tests. These two properties imply that the sum of elements of matrix X in every column is zero, i.e.,

$$\sum_{i=1}^{M} x_{ij} = 0, \qquad \text{for all } j,\ j = 1, 2, \ldots, m$$

and the inner product of any two columns is zero

$$\sum_{a=1}^{M} x_{ai} x_{aj} = 0, \qquad \text{for } i,\ j = 1, 2 \ldots, m; \quad i \neq j.$$

Thus, we have

$$X^{\mathrm{T}} X = \operatorname{diag}\left[\frac{2M}{3}, \ldots, \frac{2M}{3}\right] \in R^{m \times m}$$

where it can easily be verified that

$$M = \frac{3}{2} \sum_{i=1}^{M} x_{ij}^2$$

for every $j = 1, \ldots, m$.

For small values of δ_j, we can express the cost function using linear approximation in $[\eta_j^0 - \delta_j, \eta_j^0 + \delta_j]$ as

$$\begin{aligned} J &= J(\eta_1, \eta_2, \ldots, \eta_m) \\ &= \theta_1\left(\eta_1 - \eta_1^0\right) + \theta_2\left(\eta_2 - \eta_2^0\right) + \cdots + \theta_m\left(\eta_m - \eta_m^0\right) + \epsilon \\ &= \sum_{j=1}^{m} \theta_j(\eta_j - \eta_j^0) + \epsilon \end{aligned}$$

where θ_j are coefficients to be determined and ϵ is the error using linear approximation. From each cycle of tests according to the orthogonal array, we obtain the following M results:

$$J_i = \sum_{j=1}^{m} \theta_j \zeta_{ij} + \epsilon_i, \qquad i = 1, 2, \ldots, M \tag{9}$$

where ζ_{ij} is determined by the orthogonal array, i.e.,

$$\zeta_{ij} = \eta_j^0 + x_{ij}\delta_j. \tag{10}$$

From (9) and (10), we have

$$J_i = \sum_{j=1}^{m} \theta_j \left(\eta_j^0 + x_{ij}\delta_j\right) + \epsilon_i = \sum_{j=1}^{m} \theta_j^* x_{ij} + J_0 + \epsilon_i \tag{11}$$

where $\theta_j^* = \theta_j \delta_j$ and

$$J_0 = \sum_{j=1}^{m} \theta_j \eta_j^0.$$

The least squares solution from (11) for $\theta^* = [\theta_1^*, \ldots, \theta_n^*]^{\mathrm{T}}$ can be obtained as

$$\hat{\theta}^* = (X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}\tilde{J} \tag{12}$$

where $\tilde{J} = [J_1 - J_0, J_2 - J_0, \ldots, J_M - J_0]^{\mathrm{T}}$ and $X = [x_{ij}] \in R^{M\times m}$ (defined earlier). In this case, when $\theta^* = \hat{\theta}^*$, the least squares estimate of the function J becomes

$$\hat{J} = \sum_{j=1}^{m} \hat{\theta}_j^* \omega_j$$

where

$$\omega_j = \frac{\eta_j - \eta_j^0}{\delta_j}.$$

Therefore,

$$\begin{aligned} \hat{J} &= \sum_{j=1}^{m} \hat{\theta}_j^* \frac{\eta_j - \eta_j^0}{\delta_j} \\ &= \frac{\hat{\theta}_1^*}{\delta_1}\left(\eta_1 - \eta_1^0\right) + \frac{\hat{\theta}_2^*}{\delta_2}\left(\eta_2 - \eta_2^0\right) + \cdots + \frac{\hat{\theta}_m^*}{\delta_m}\left(\eta_m - \eta_m^0\right). \end{aligned}$$

We can then obtain the gradient of $\hat{J}$ as

$$\nabla \hat{J} = \left\{ \frac{\hat{\theta}_1^*}{\delta_1}, \frac{\hat{\theta}_2^*}{\delta_2}, \ldots, \frac{\hat{\theta}_m^*}{\delta_m} \right\}.$$

Since $X^{\mathrm{T}}X$ is diagonal, from (12), we have

$$\begin{aligned} \hat{\theta}_j^* &= \frac{3}{2M} \sum_{i=1}^{M} x_{ij}(J_i - J_0) \\ &= \frac{3}{2M}\left(V_j^{(3)} - V_j^{(1)}\right), \qquad j = 1, 2, \ldots, m \end{aligned} \tag{13}$$

where $\{V_j^{(k)}, k = 1, 3\}$ indicates the total contribution of the kth level of the factor η_j to the cost function in a cycle of tests [see examples in (3)]. We can see that the gradient of the cost function is determined by the difference between the total contribution of the upper level, i.e., $k = 3$, and the lower level, i.e., $k = 1$. Equation (13) shows that Fig. 1(a), (b), (d), and (e) can be used directly for estimating the gradient of the cost function J as is used in the present algorithm for multiuser detection.

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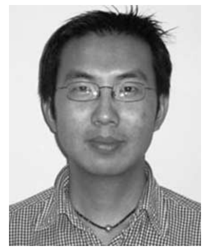

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