Book Review

Control Systems with Actuator Saturation: Analysis and Design—Tingshu Hu and Zongli Lin (Boston, MA: Birkhäuser, 2001). *Reviewed by Derong Liu*

Research in control systems has long been recognized to have practical origins. One such example is the subject of the book: control systems with saturating actuators. A recent survey article [1] on this topic elaborated that *"the control engineer armed with linear analysis and design techniques soon encounters a fundamental nonlinear problem that threatens the operation of otherwise sound designs: unexpectedly large amplitude disturbances can push a system's actuators into saturation, thus forcing the system to operate in a mode for which it was not designed and from which it may not be able to recover."* Earlier works related to actuator saturation have dealt with control systems with constrained or bounded controls (e.g., [5], [6], and [8]–[10]). The book, together with several other books [3]–[5], [7], represents the most recent development in dealing with such a practically imperative and yet theoretically challenging problem.

All control actuation devices are subject to amplitude saturation. Every conceivable physical control input in every conceivable application is ultimately limited. Examples of such control input include force, torque, thrust, stroke, voltage, current, and flow rate. In fact, the importance of actuator saturation was recognized and reflected in the original formulation of many fundamental control problems, including controllability and optimal control. Control problems that involve hard nonlinearities such as actuator saturation, however, turn out to be difficult to deal with. As a result, even though there have been continual efforts in addressing actuator saturation (see the chronological bibliography [1] and the references cited in the present book), its effects have not been addressed thoroughly in modern control literature.

The book by Hu and Lin examines some fundamental control problems in a systematic manner. Unlike [4] and [7] which treat semistable systems (a linear system is said to be semistable if none of its poles lie in the open right-half plane), it deals with general systems that could be strictly unstable. The book starts with an explicit description of the null controllable region: the set of states that can be driven to the origin in a finite time by a bounded control delivered by the saturating actuators. It is important to note that, for a semistable system that is controllable in the usual linear sense, the null controllable region is simply the entire state–space. With the null controllable region explicitly described, the book proceeds to address various control problems within the null controllable region.

Chapter 1 provides an introduction to the book. A brief discussion on the problem of actuator saturation and the state-of-the-art in addressing the problem leads to the following two fundamental questions to which the entire book is devoted.

- For a general, not necessarily semistable, linear system with saturating actuators, what is the null controllable region?
- How does one design feedback laws that work over the entire asymptotically null controllable region or a large portion of it?

The first major theme of the book concerns null controllability, which is covered in Chapters 2 and 3. Chapter 2 gives explicit descriptions of the null controllable region of a continuous-time linear system with the bounded controls delivered by saturating actuators. The boundaries of the null controllable region are expressed both by analytical formulas and as trajectories of the time-reversed system under certain bang-bang type controls. Several examples are worked out to illustrate the shapes of null controllable regions. Chapter 3 is the discrete-time counterpart of Chapter 2 for continuous-time systems.

The second major theme of the book concerns the stabilizability over the null controllable region which is addressed in Chapters 4, 5, and 6. Chapters 4 and 5 study the stabilizability at the origin of linear systems with saturating actuators. Stabilizing feedback laws are designed to produce a domain of attraction that is arbitrarily close to the null controllable region. Such a stabilization problem is referred to as semiglobal stabilization over the null controllable region. Chapter 4 deals with continuous-time systems, while Chapter 5 deals with discrete-time systems. Chapter 6 considers continuous-time linear systems that are subject to both actuator saturation and input-additive bounded disturbances. Feedback laws are constructed that cause all trajectories starting from within any *a priori* specified (arbitrarily large) compact subset of the null controllable region to converge to another *a priori* specified (arbitrarily small) neighborhood of the origin. Such a design problem is referred to as semiglobal practical stabilization over the null controllable region.

Control design for linear systems with saturating actuators using optimization-based methods is the third major theme of the book which is investigated in Chapters 7–11. Analysis and design objectives are formulated as optimization problems which are solved using a linear matrix inequality (LMI) approach. LMI-based approaches are very popular in the study of control systems with actuator saturation (see, e.g., [3]). A remarkable feature of the results presented in the book is that the optimization problems are convex and can be exactly transformed into LMIs. This is made possible by some innovative tools developed by the authors. These tools not only simplify the optimization problems, but also lead to less conservative results. Chapter 7 is devoted to the estimation of the domain of attraction under a saturated linear feedback, using invariant ellipsoids. It first introduces a powerful tool which embeds the saturated feedback control into the convex hull of a group of linear feedback controls. Using this tool, some criteria for the invariance of ellipsoids are established. These criteria are shown to be less conservative than the existing criteria, e.g., the circle criterion and the vertex criterion. More importantly, the conditions of the criteria are equivalent to LMIs and, hence, the optimization problem of maximizing the invariant ellipsoids can easily be solved. This analysis method is then utilized in Chapter 8 to arrive at a method for designing linear state feedback laws that will result in the largest estimated domain of attraction. Each of these two chapters treats both continuous-time and discrete-time systems. Chapter 9 develops a design method for arriving at simple nonlinear feedback laws that achieve semiglobal stabilization on the null controllable region and, at the same time, guarantee regional performance. Both continuous- and discrete-time systems are considered. Chapter 10 addresses the problem of controlling linear systems subject to both actuator saturation and disturbances. Unlike Chapter 6, here the disturbances are not input additive and can enter the system at any point. Design problems that capture both large domains of attraction and strong disturbance rejection capability are formulated and solved as optimization problems with LMI constraints. Both continuous- and discrete-time systems are considered. Chapter 11 examines

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Digital Object Identifier 10.1109/TAC.2003.813146

the problem of maximizing the convergence rate inside a given ellipsoid for both continuous- and discrete-time systems subject to actuator saturation. Simple methods are also proposed for determining the largest ellipsoid of a given shape that can be made invariant with a saturated control. For continuous-time systems, the maximal convergence rate is achieved by a bang-bang type control with a simple switching scheme. A suboptimal convergence rate can be achieved with saturated high-gain linear feedback. For discrete-time systems, the maximal convergence rate is achieved by a coupled saturated linear feedback.

The fourth major theme of the book deals with the problem of output regulation. Chapters 12 and 13 formulate and solve the classical problem of output regulation for continuous- and discrete-time linear systems with saturating actuators. The problem is to design stabilizing feedback laws that, in the presence of disturbances, cause the plant output to track reference signals asymptotically. Both the reference signals and the disturbances are modeled by a reference system, called the exosystem. The asymptotically regulatable region, the set of all initial conditions of the plant and the exosystem for which the output regulation is possible, is characterized. Feedback laws that achieve output regulation on the asymptotically regulatable region are constructed.

Finally, Chapter 14 summarizes results on the analysis and design of linear systems subject to sensor and state saturation. In particular, it is established that a linear stabilizable and detectable system can be semiglobally stabilized by linear feedback of the saturated output measurement as long as the system in the absence of output saturation does not have any pole in the open right-half plane. The stability properties of both continuous- and discrete-time linear planar systems with saturation nonlinearities are analyzed. Necessary and sufficient conditions for global asymptotic stability of such systems are established. For the continuous-time systems considered in this chapter, the saturation is imposed on the derivative of the state, which is equivalent to a saturation nonlinearity acting on the state vector itself. In this case, the state trajectories of such a system may not be bounded. This is in contrast to the systems considered in [2] and [5], where the derivative of the state is set to zero whenever the state saturates, resulting in bounded state trajectories. On the other hand, for the discrete-time systems considered here, the saturation is imposed directly on the state and, thus, can be viewed as the discrete-time counterpart of the continuous-time systems considered in [2]. Thus, the results in this concluding chapter, along with those of [2] and [5], provide a good illustration of the complexity and subtlety embedded in seemingly simple linear systems coupled with saturation nonlinearities.

The book reads very well. The topics chosen are fundamental control problems and are all treated in great depth. Results presented in the book represent a significant contribution to the important research area of control systems with actuator saturation. The results, along with an extensive bibliography, will prove to be an important resource for researchers in this area as well as researchers interested in entering this area. In this regard, I would have liked to see the authors include a chapter discussing future research topics. It is also noted that many results developed in the book may have applications in the study of a class of recurrent artificial neural networks [5].

The book will prove equally valuable to practicing engineers. The results included in the book are applicable to general systems, including strictly unstable ones. Although the mathematics involved are on the heavy side, all of the analysis and design algorithms are explicit, and the optimization problems involved are convex and can readily and reliably be solved using the Matlab LMI Toolbox. Moreover, all of the results are illustrated with fully worked out examples and simulations.

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