INTERNATIONAL JOURNAL OF ROBUST AND NONLINEAR CONTROL
Int. J. Robust Nonlinear Control 2004; **14**:925–927 (DOI: 10.1002/rnc.833)

BOOK REVIEW

CONTROL SYSTEMS WITH ACTUATOR SATURATION: ANALYSIS AND DESIGN by T. Hu and Z. Lin; Birkhäuser, Boston, 2001, xvi + 392pp., ISBN 0-8176-4219-6

This book from Birkhäuser joins the recent monographs [2, 4] and the edited volume [3] on the subject of control systems with actuator saturation, a research topic that practically imperative and yet theoretically challenging. To quote from a recent survey on the topic by Bernstein and Michel [1], *control engineers armed with linear analysis and design techniques soon encounter a fundamental nonlinear problem that threatens the operation of otherwise sound designs*: *unexpectedly large amplitude disturbances can push a system's actuators into saturation*, *thus forcing the system to operate in a mode for which it was not deigned and from which it may not be able to recover*. The prospects for failure are both pervasive and frightening [5]. All control actuation devices are subject to amplitude saturation. Force, torque, thrust, stroke, voltage, current, flow rate, and every conceivable physical input in every conceivable application of control technology is ultimately limited. In fact, the importance of actuator saturation was recognized and reflected in the original formulation of many fundamental control problems, including controllability and time optimal control. Control problems that involve hard nonlinearities such as actuator saturation, however, turn out to be difficult to deal with. As a result, even though there have been efforts in addressing actuator saturation (see the chronological bibliography [4] and the references cited in the present book), its effects have been ignored in most of the modern control literature.

The book by Hu and Lin examines some fundamental control problems in a systematic manner. Unlike [2] and [4] which treated semi-stable systems (a linear system is said to be semi-stable if none of its poles lie in the open right half plane), it deals with general systems that could be strictly unstable. The book starts with an explicit description of the null controllable region: the set of states that can be driven to the origin in a finite time by a bounded control delivered by saturating actuators. It is important to note that, for a semi-stable system that is controllable in the usual linear sense, the null controllable region is simply the entire state space. With the null controllable region explicitly described, the book proceeds to address various control problems within the null controllable region. These problems include stabilizability, transient performance, disturbance rejection, and output regulation. Analysis tools are also provided for performance assessment of existing control systems under actuator saturation. More specifically, the book is organized into the following fourteen chapters.

Chapter 1 provides introduction to the book. A brief discussion on the problem of actuator saturation and the state-of-the-art in addressing the problem leads to the following two fundamental questions, around which Chapters 2–13 of the book will be presented:

- For a general, not necessarily semi-stable, linear system with saturating actuators, what is the null controllable region?
- How to design feedback laws that work on the entire asymptotically null controllable region or a large portion of it?

Some notation and acronyms that are used throughout the book are also defined in Chapter 1.

Chapter 2 gives explicit descriptions of the null controllable region of a continuous-time linear system with the bounded controls delivered by saturating actuators. The boundaries of the null controllable region are expressed both in analytical formulae and as trajectories of the time-reversed system under certain bang–bang type controls. Several examples are worked out to illustrate the shapes of null controllable regions. Chapter 3 is the discrete-time counterpart of Chapter 2 on continuous-time systems.

Chapters 4 and 5 study the stabilizability at the origin of linear systems with saturating actuators. Stabilizing feedback laws are designed to produce a domain of attraction that is arbitrarily close to the null controllable region. Such a stabilization problem is referred to as semi-global stabilization on the null controllable region. Chapter 4 deals with continuous-time systems, and Chapter 5 deals with discrete-time systems.

Chapter 6 considers continuous-time linear systems that are subject to both actuator saturation and input-additive bounded disturbance. Feedback laws are constructed that cause all trajectories starting from within any *a priori* specified (arbitrarily large) compact subset of the null controllable region to converge to another *a priori* specified (arbitrarily small) neighbourhood of the origin. Such a design problem is referred to as semi-global practical stabilization on the null controllable region.

Chapters 7–11 investigate the problem of controlling a linear system with saturating actuators from a different angle. Analysis and design objectives are formulated into optimization problems which are solved with linear matrix inequality (LMI) approach. LMI-based approaches are very popular in the study of control systems with actuator saturation (see, e.g., [3], where six out of the 11 papers uses LMI-based approaches). A remarkable feature of the results presented in the present book is that the optimization problems are convex and can be exactly transformed into LMIs. This is made possible by some innovative tools developed by the authors. These tools not only simplify the optimization problems, but also lead to less conservative results.

Chapter 7 is devoted to the estimation of the domain of attraction under a saturated linear feedback, by using invariant ellipsoids. It first introduces a powerful tool which places the saturated feedback control in the convex hull of a group of linear feedback controls. Using this tool, some criteria for the invariance of ellipsoids are established. These criteria are shown to be less conservative than the existing criteria, e.g. the circle criterion and the vertex criterion. More importantly, the conditions of the criteria are equivalent to LMIs and hence the optimization problem of maximizing the invariant ellipsoids can be easily solved. This analysis method is then utilized in Chapter 8 to arrive at a method for designing linear state feedback laws that would result in the largest estimated domain of attraction. Each of these two chapters treats both continuous-time and discrete-time systems.

Chapter 9 develops a design method for arriving at simple nonlinear feedback laws that achieve semi-global stabilization on the null controllable region and, in the mean time, guarantee regional performance. Both continuous-time and discrete-time systems are considered.

Chapter 10 addresses the problem of controlling linear systems subject to both actuator saturation and disturbance. Unlike Chapter 6, here the disturbance is not input additive and can enter the system from anywhere. Design problems that capture both large domains of attraction and strong disturbance rejection capability are formulated and solved as optimization problems with LMI constraints. Both continuous-time and discrete-time systems are considered.

Chapter 11 examines the problem of maximizing the convergence rate inside a given ellipsoid for both continuous-time and discrete-time systems subject to actuator saturation. Simple methods are also proposed for determining the largest ellipsoid of a given shape that can be made invariant with a saturated control. For continuous-time systems, the maximal convergence rate is achieved by a bang–bang type control with a simple switching scheme. A sub-optimal convergence rate can be achieved with saturated high-gain linear feedback. For discrete-time systems, the maximal convergence rate is achieved by a coupled saturated linear feedback.

Chapters 12 and 13 formulate and solve the classical problem of output regulation for continuous-time and discrete-time linear systems with saturating actuators. The problem is to design stabilizing feedback laws that, in the presence of disturbances, cause the plant output to track reference signals asymptotically. Both the reference signals and the disturbances are modeled by a reference system, called the exosystem. The asymptotically regulatable region, the set of all initial conditions of the plant and the exosystem for which the output regulation is possible, is characterized. Feedback laws that achieve output regulation on the asymptotically regulatable region are constructed.

Finally, Chapter 14 summarizes some results on the analysis and design of linear systems subject to sensor saturation and state saturation. In particular, it is established that a linear stabilizable and detectable system can be semi-globally stabilized by linear feedback of the saturated output measurement as long as the system in the absence of output saturation does not have any pole in the open right-half plane. The stability properties of both continuous-time and discrete-time planar systems under state saturation are fully analysed. The long standing issue of the necessary and sufficient conditions for global asymptotic stability of these systems is resolved.

The book is very well written. The topics chosen are all fundamental control problems and are treated in depth. All the results presented in the book appear to be the authors' own recent research findings and, without doubt, represent a significant contribution to the important research topic of analysis and design for control systems with actuator saturation. The results included in the book, along with an extensive bibliography will prove to be an important resource for researchers in this area as well as researchers interested in entering this area. In this regard, I would have liked to see the authors included a chapter discussing future research topics.

The book will prove equally valuable to practicing engineers. The results included in the book are applicable to general systems including strictly unstable ones. Although the mathematics involved in the book is on the heavy side, all the analysis and design algorithms are explicit. All optimization problems involved are convex and can readily and reliably be solved with the Matlab LMI Toolbox. Moreover, all the results are illustrated with fully worked out examples and simulation.

In conclusion, the book will be a valuable addition to the libraries of both control theory researchers and control engineering practioners.

REFERENCES

1. Bernstein DS, Michel AN. A chronological bibliography on saturating actuators. *International Journal of Robust and Nonlinear Control* 1995; **5**:375–380.
2. Lin Z. *Low Gain Feedback*. Lecture Notes in Control and Information Sciences, Vol. 240. Springer: London, 1998.
3. Kapila V, Grigoriadis KM (eds.). *Actuator Saturation Control*. Marcel Dekker: New York, 2002.
4. Saberi A, Stoorvogel A, Sannuti P. *Output Regulation and Control Problems with Regulation Constraints*. Springer: London, 1999.
5. Stein G. Respect the unstable, Bode Lecture. *IEEE Conference on Decision and Control*. Tampa, FL, 1989.

Derong Liu
Department of Electrical and Computer Engineering, University of Illinois at Chicago
Chicago, IL 60607, USA
E-mail: dliu@ieee.org

